

SCENARIOS OF OPTIMAL CONTROL OF TRANSREGIONAL MIGRATION PROCESSES UNDER RISK

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Scenarios of migration are considered as applied to the problem of optimal control of transregional migration with zero risk of overpopulation or insufficient population and migration with obtaining the minimum of the objective functional and minimizing risk. Numerical algorithms for the scenarios considered are developed. The algorithms are implemented in an applied geoinformation decision-support system for control of transregional migration of population.

Keywords: *optimal control, discrete optimization, migration, utility function, dynamic system, demographic risk.*

INTRODUCTION

Scenarios of optimal control of transregional migration processes that are presented in this paper are specific to the optimal control problem $A2$ from [1] and are based on models of the two-level hierarchical organizational system [2–4] of optimal control for linear systems of ordinary differential equations [5–7] and models of the utility function for demographic processes [8]. The general algorithm proposed in [1] for solution of this problem in the most general statement leads to a time-consuming discrete optimization problem whose solution lies in finding the global extremum of a function of a large number N_{tot} (that amounts to about several hundreds) of variables. For practical operation of geoinformation application systems in real-time, individual scenarios should be considered that, on the one hand, form a modified class of subtasks with additional constraints and, on the other hand, require the development of more efficient methods (algorithms) of searching for optimal solutions. In this work, we present (1) the scenario of absence of demographic risk at the initial moment of time with preservation of this condition over the whole territory at subsequent moments of time with the minimization of the objective functional of population distribution over given domains; (2) the scenario of a given population distribution over chosen regions (oblasts) of the domain being considered at the initial and subsequent moments of time with the minimization of the objective functional of total demographic risk in the other regions of the oblast. New algorithms are developed for the directed search for solution as applied to these scenarios (optimal control problems formulated in a new form) on the basis of analyzing gradients of objective functionals, taking into account the specificity of the domain of minimization, and using the methodology of sequential analysis of variants [9]. In this article, the results of functioning of the algorithms implemented in a geoinformation support system of making decisions on optimal control of transregional migration processes under risk are illustrated by an example of socioeconomic and ecological data for Ukraine over 2001–2004.

STATEMENT OF THE GENERAL PROBLEM OF OPTIMAL CONTROL OF MIGRATION PROCESSES UNDER RISK

Let us consider an optimal control model for the speed problem specified on the basis of a linear system of ordinary differential equations. We introduce the dependence of the parameters of the system on time t , on the identification parameter

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of a region of the society (state) $\omega \in \Omega$ (the number of the region can be used as such a parameter) for a finite discrete set that specifies the area of investigation Ω , on the number of a social group $k \in K_0 \subset N$, and on socioeconomic and ecological parameters x of the system. Within the framework of a two-level hierarchical system Center C_0 , a bounded collection of dynamic control parameters is used, namely, a vector function $g(t) = (g_1(t), \dots, g_n(t))$ for the determination of dynamic controlled socioeconomic parameters (trajectories of a process $x(t, \omega, k, g) = (x_1(t, \omega, k, g), \dots, x_n(t, \omega, k, g))$) with a view to coordinating social groups C_k . To form a hypothesis on the behavior of a group C_k , a model of the scalar utility function $u(t, x, \omega, k)$ is used that is the life quality index of the social group in a region ω at a moment of time t with given parameters $x = x(t, \omega, k, g)$. It functions as a binary relation and determines the advantage (equivalence) of the first collection of parameters $(t, x, \omega, k)^{(1)}$ over the second collection $(t, x, \omega, k)^{(2)}$ if we have $u(t, x, \omega, k)^{(1)} \geq u(t, x, \omega, k)^{(2)}$. The hypothesis of optimal behavior of the group C_k at a fixed moment of time t is specified with the help of an empirical function of migration balance $M = M(u(t, x, \omega, k, g))$ that represents the difference between the numbers of citizens who arrive at and depart from the region during a definite time period. We introduce the following designations: $D(t, x, \omega, k, g)$ denotes the population size, $\tilde{D}(T_0, \omega, k)$ is the desirable population size in regions $\omega \in \Omega_1$: $\Omega_1 \neq \emptyset$, $\Omega_2 = \Omega \setminus \Omega_1 \neq \emptyset$ that is established by the Center C_0 , D_0 is the initial population size, R is the balance between birth and death rates during a definite time period (a prescribed step function), M_E is the balance of abroad migration during a definite time period (a prescribed function), $\chi(x)$ is the Heaviside function; coefficients α_0 and α_k , an odd parameter $q \in N$, a parameter j_0 , and a weight coefficient $\gamma_l(k) \neq 0$ are parameters specified by experts (when $\gamma_l > 0$, the effect of the parameter x_l on the migration process is positive and on the emigration process is negative, and when $\gamma_l < 0$, this effect is inverse); \tilde{x}_l and \hat{x}_l are expert estimates of the "inertness" of the k th social group with respect to the parameter x_l ; A is a given linear continuous operator ($R^n \rightarrow R^n$), $r(t, \omega, k, g)$ is the value of the risk of overpopulation or insufficient population that belongs to a social group C_k and lives in the oblast ω at the moment t , $\tilde{D}(t, \omega, k)$ and $\hat{D}(t, \omega, k)$ are given threshold values of the population size, and $\varepsilon > 0$ is an error. We formulate the problem of optimal control [1] in the form

$$G^* = \text{Arg min}_{g \in G} (J_1(T_0, k, g) + J_2(T_0, k, g)), \quad (1)$$

$$J_1(T_0, k, g) = \sum_{\omega \in \Omega_1} \lambda_1(\omega) \left[\tilde{D}(T_0, \omega, k) - D(T_0, x, \omega, k, g) \right]^2,$$

$$J_2(T_0, k, g) = \sum_{\omega \in \Omega} \lambda_2(\omega) r(T_0, \omega, k, g),$$

$$\dot{D}(t, x, \omega, k, g) = M(u(t, x, \omega, k, g)) + M_E(t, \omega, k) + R(t, \omega, k),$$

$$D(t_0, \omega, k) = D_0(\omega, k),$$

$$\dot{x}(t, \omega, k) = Ag, \quad x(t_0, \omega, k) = x^0(\omega, k),$$

$$x \in X(t) \subset R^n, \quad X(t) = X_1(t) \times \dots \times X_n(t),$$

$$g \in G \subset R^n, \quad g^* \in G^* \subset G, \quad t \in [t_0, T_0], \quad \omega \in \Omega,$$

$$M(u(t, x, \omega, k, g)) = \alpha_0 + \sum_{j=1}^{j_0} \alpha_j (u(t, x, \omega, k, g))^{2j-1}, \quad \alpha_j \geq 0 \quad (j = \overline{1, j_0}),$$

$$u(t, x, \omega, k) = \sum_{l=1}^n \gamma_l(k) ((x_l(t, \omega, k, g) - \tilde{x}_l(k)) \chi(\tilde{x}_l(k) - x_l(t, \omega, k, g)) + (x_l(t, \omega, k, g) - \hat{x}_l(k)) \chi(x_l(t, \omega, k, g) - \hat{x}_l(k)))^q,$$

$$r(t, x, \omega, k, g) = \frac{1}{t - t_0} \int_{t_0}^t \{ (\tilde{D}(t, \omega, k) - D(M(t, x, \omega, k, g))) \chi(\tilde{D} - D(M(t, x, \omega, k, g)))$$

$$+ (D(M(t, \omega, k, g)) - \hat{D}(t, \omega, k)) \chi(D(M(t, \omega, k, g)) - \hat{D}(t, \omega, k)) \}^2 dt + r_0(\omega, k),$$

$$\sum_{\omega \in \Omega} M(u(t, x, \omega, k, g)) = 0, \quad \hat{G}_0^* = \text{Arg} \min_{\hat{g}^* \in \hat{G}} (T(k, \hat{g}^*) - t_0),$$

$$\left| J_1(T, k, \hat{g}^*) - J_1(T_0, k, g^*) \right| \leq \varepsilon, \quad T \in [t_0, T_0]$$

$$\hat{G}^* = \left\{ \hat{g}^*(t) \mid \hat{g}^*(t) = \begin{cases} g^*(t), & t \in [t_0, T], \\ 0, & t \in (T, T_0], \end{cases} \quad g^*(t) \in G^* \right\},$$

where $\lambda_1(\omega)$ and $\lambda_2(\omega)$ are positive dimensionless weight coefficients. The solution of problem (1) is a set of optimal controls $\hat{g}^{(0)*} \in \hat{G}_0^* \subset G \subset R^n$. The conditions of Theorem 2 from [1] are considered to be true since g and its derivative g_t are bounded within $L_2([t_0, T_0])$, whence the compactness of the set G follows.

SCENARIO OF AN OPTIMAL CONTROL WITH ZERO RISK

For problem (1), we consider the case when $A = E$ is a unit operator. This scenario is based on the following additional constraints on the parameters of the problem:

$$r_0(\omega, k) = 0,$$

$$W(t, \omega, k) = (D(M(t, x, \omega, k, g)) - \tilde{D}(t, \omega, k))\chi(\tilde{D} - D(M(t, x, \omega, k, g)))$$

$$+ (D(M(t, x, \omega, k, g)) - \hat{D}(t, \omega, k))\chi(D(M(t, x, \omega, k, g)) - \hat{D}(t, \omega, k)) = 0,$$

$$t \in [t_0, T_0], \quad \omega \in \Omega, \quad k \in K_0, \quad g \in G. \quad (2)$$

Constraints (2) require that risk be equal to zero at the initial moment of time and that these conditions be preserved during each subsequent moment of time over the entire given territory. Taking into account the nonnegativity of the functionals J_1 and J_2 , condition (2) practically means that the Center C_0 considers a scenario of development of the migration process for the case when the absence of demographic risk is more preferable (the functional J_2) than the attainment of an optimal distribution of social groups among regions (the functional J_1), i.e., the weight coefficient $\lambda_1 \ll \lambda_2$. Condition (2) forms a new domain of admissible system trajectories and, accordingly, a domain of admissible controls that is determined by the following inequalities:

$$P_1(t, \omega, k) \leq \int_{t_0}^t \sum_{j=1}^{j_0} \alpha_j \left(\sum_{l=1}^n \gamma_l(k) ((x_l(\tau, \omega, k, g) - \tilde{x}_l(k))\chi(\tilde{x}_l(k) - x_l(\tau, \omega, k, g)) + (x_l(\tau, \omega, k, g) - \tilde{x}_l(k))\chi(x_l(\tau, \omega, k, g) - \tilde{x}_l(k)))^q \right) d\tau \leq P_2(t, \omega, k),$$

$$P_1(t, \omega, k) = \tilde{D}(t, \omega, k) - D_0(t_0, \omega, k) - \alpha_0(t - t_0) - \int_{t_0}^t (M_E(\tau, \omega, k) + R(\tau, \omega, k)) d\tau,$$

$$P_2(t, \omega, k) = \hat{D}(t, \omega, k) - D_0(t_0, \omega, k) - \alpha_0(t - t_0) - \int_{t_0}^t (M_E(\tau, \omega, k) + R(\tau, \omega, k)) d\tau.$$

Since control is considered on the class of bounded step functions (the compactness of the set G) [1], and condition (3) determines a compact set of optimal controls $g \in \tilde{G} \subset G$. Problem (1), (2) is correctly stated. To construct a numerical

algorithm, we consider the covering of the segment $[t_0, T_0]$ by a finite set of identical segments $[t_{p-1}, t_p]$, $\tau_0 = t_p - t_{p-1}$, $p = \overline{1, P_0}$, $t_{P_0} = T_0$. Control is considered on the class of bounded step functions

$$g^{(0)}(t, \omega, k) = a_0(t, \omega, k) = \sum_{p=1}^{P_0} (\chi(t - t_{p-1}) - \chi(t - t_p)) g_p(\omega, k),$$

$$\check{L}_{lp}(\omega, k) \leq g_{lp}(\omega, k) \leq \widehat{L}_{lp}(\omega, k), \quad (4)$$

where $\check{L}_{lp}(\omega, k) \leq 0 \leq \widehat{L}_{lp}(\omega, k)$ are constants specified by experts for $p = \overline{1, P_0}$, $l = \overline{1, n}$, $\omega \in \Omega$, $k \in K_0$. The existence at least one optimal control satisfying condition (2) over the class of step functions follows from the form of constraints (4) for the compact set G and zero initial conditions for risks. Speed problem (1), (2) can be formulated in terms of discrete optimization theory [9]. We represent the functional of the population size in the additive form

$$D(t_p, \omega, k, g) = D_0(\omega, k) + \tau_0 (P\alpha_0 + \sum_{p=1}^P (M_E(t_p, \omega, k) + R(t_p, \omega, k)))$$

$$+ \sum_{p=1}^P \sum_{j=1}^{j_0} \alpha_j \int_{t_{p-1}}^{t_p} \left(\sum_{l=1}^n \gamma_l(k) \left(((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) - \check{x}_l(k)) \right. \right.$$

$$\times \chi((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) - \check{x}_l(k))$$

$$+ ((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) - \widehat{x}_l(k))$$

$$\left. \left. \times \chi((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) - \widehat{x}_l(k)) \right)^q \right) dt,$$

$$x_l(t_p, \omega, k, g_{lp}) = \tau_0 \sum_{i=1}^p g_{li}(\omega, k) + x_l^0(\omega, k),$$

$$g_0(\omega, k) = (g_{10}(\omega, k), \dots, g_{n0}(\omega, k)) = 0.$$

By analogy with the methodology of sequential analysis of variants, for each segment $[t_{p-1}, t_p]$ ($p = \overline{1, P_0}$), we specify a collection of control parameters $g \in \check{G} \subset G$ that provide the minimum of the functional

$$g_p^*(\omega, k) = (g_{1p}^*(\omega, k), \dots, g_{np}^*(\omega, k)) = \arg \inf_{g_p \in \check{G}} (J_1(t_p, k, g_p)) \quad (6)$$

successively for each $p = 1, 2, \dots$. Computations come to an end after the attainment of the minimum of functional (6) with a given accuracy $\varepsilon > 0$ when $p = p^*$ or a final moment of time when $p = P_0$. Using the theorem on the differentiation of an integral with respect to a parameter, we determine the gradient of the functional J_1 with the help of the function of the population size from functional (5) in the segment $[t_0, t_p]$ as follows:

$$\frac{\partial J_1(t_p, k, g)}{\partial g_{lp}^*(\omega, k)} = 2\lambda_1(\omega) [\widehat{D}(T_0, \omega, k) - D(t_p, \omega, k, g)] \frac{\partial D(t_p, \omega, k, g)}{\partial g_{lp}^*(\omega, k)}, \quad (7)$$

$$\frac{\partial D}{\partial g_{lp}^*(\omega, k)} = \gamma_{l^*}(k) \sum_{j=1}^{j_0} \alpha_j \int_{t_{p-1}}^{t_p} \left(\sum_{l=1}^n \gamma_l(k) \left(((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) \right. \right.$$

$$- \check{x}_l(k)) \chi((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) - \check{x}_l(k)) + ((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) - \widehat{x}_l(k))$$

$$\begin{aligned}
& \times \chi((\tau - t_{p-1})g_{lp} + x_l(t_{p-1}, \omega, k, g_{lp-1}) - \tilde{x}_l(k)) \Big)^q \Big)^{2j-2} \\
& \times (2j-1)q(((\tau - t_{p-1})g_{l^*p} + x_{l^*}(t_{p-1}, \omega, k, g_{l^*p-1}) - \tilde{x}_{l^*}(k))\chi((\tau - t_{p-1})g_{l^*p} \\
& + x_{l^*}(t_{p-1}, \omega, k, g_{l^*p-1}) - \tilde{x}_{l^*}(k)) + ((\tau - t_{p-1})g_{l^*p} + x_{l^*}(t_{p-1}, \omega, k, g_{l^*p-1}) \\
& - \tilde{x}_{l^*}(k))\chi((\tau - t_{p-1})g_{l^*p} + x_{l^*}(t_{p-1}, \omega, k, g_{l^*p-1}) - \tilde{x}_{l^*}(k)))^{q-1} \\
& \times (\tau - t_{p-1})(\chi((\tau - t_{p-1})g_{l^*p} + x_{l^*}(t_{p-1}, \omega, k, g_{l^*p-1}) - \tilde{x}_{l^*}(k)) \\
& + \chi((\tau - t_{p-1})g_{l^*p} + x_{l^*}(t_{p-1}, \omega, k, g_{l^*p-1}) - \tilde{x}_{l^*}(k)))dt.
\end{aligned} \tag{8}$$

Since the exponents $(2j-2)$ and $(q-1)$ are even and coefficients $\alpha_j \geq 0$ ($j = \overline{1, j_0}$) and $\tau \geq t_{p-1}$, the subintegral function in formula (8) is nonnegative. It follows from formulas (7) and (8) that the sign of the l^* -component of the gradient of the functional J_1 is determined by the sign of $(\tilde{D}(T_0, \omega, k) - D(t_p, x, \omega, k, g))$ and the sign of the parameter $\gamma_{l^*}(k)$. In fact, this means that the optimal values of the l^* -component of the system trajectory $x_{l^*}(t_p)$ are in the segment connecting the initial value of the system trajectory with the projection of a given point onto the boundary of the compact domain determined by the constraints of problem (1) and inequalities (3). But the components of the gradient of the functional can assume zero values not only in the domain of the minimum of the functional but also in the domain of inertness of the utility function (the presence of the Heaviside function under the integration sign). Therefore, along with formulas (7) and (8), one should also take into account the smoothness properties of the function $D(t_p, x, \omega, k, g)$ that enters in the expressions for both functionals. Let us consider a numerical algorithm for solution of problem (1). We fix $k \in K_0$ (the C_k th social group).

1. Specify the initial parameters and characteristic constants of problem (1).
2. Specify the current step with respect to time $p \geq 1$ and determine a moment of time $t_p = t_{p-1} + \tau_0$.
3. Fix the sequent region $\omega \in \Omega_1$. For this region, determine the boundary value of the population size $D_r(t_p, \omega, k)$ for zero risk as follows:

$$D_r(t_p, \omega, k) = \begin{cases} \tilde{D}(T_0, \omega, k) & \text{if } \tilde{D}(T_0, \omega, k) \in [\tilde{D}(t_p, \omega, k), \widehat{D}(t_p, \omega, k)], \\ \widehat{D}(t_p, \omega, k) & \text{if } \tilde{D}(T_0, \omega, k) > \widehat{D}(t_p, \omega, k), \\ \tilde{D}(t_p, \omega, k) & \text{if } \tilde{D}(T_0, \omega, k) < \tilde{D}(t_p, \omega, k). \end{cases}$$

4. According to formulas (7) and (8), determine the initial value of the auxiliary control parameter $\Delta_{lp}(\omega, k)$, $l = \overline{1, n}$, as follows:

$$\Delta_{lp}(\omega, k) = \begin{cases} \widehat{L}_{lp}(\omega, k)/N_\gamma & \text{if } (\gamma_l(k)(D(t_{p-1}, x, \omega, k, g) - D_r(t_p, \omega, k))) < 0, \\ \tilde{L}_{lp}(\omega, k)/N_\gamma & \text{if } (\gamma_l(k)(D(t_{p-1}, x, \omega, k, g) - D_r(t_p, \omega, k))) > 0, \\ 0 & \text{if } (\gamma_l(k)(D(t_{p-1}, x, \omega, k, g) - D_r(t_p, \omega, k))) = 0, \end{cases} \tag{9}$$

where N_γ is the initial value of the number of steps along the segment connecting the initial point of the system trajectory and the boundary point determined by the boundary value of a compact set $G \subset R^n$.

In this case, the use of the gradient of a functional according to formulas (7) and (8) is, on the one hand, sufficiently time-consuming for the numerical algorithm and, on the other hand, one should realize additional checks and execute procedures similar to (9) in the cases when all the components of the gradient of the functional equal zeros and when the system trajectory has not yet attained the minimum point (the domain of inertness of the utility function). However, formulas (5), (7), and (8) imply that the maximal value of the population size $D_{\max}(t_p, \omega)$ is attained when the system moves in the direction of increasing the values of stimulators $g_{lp}(\omega, k) = \widehat{L}_{lp}(\omega, k)$ (when $\gamma_l > 0$) to their maximum and decreasing the values of destimulators of a social system $g_{lp}(\omega, k) = \tilde{L}_{lp}(\omega, k)$ (when $\gamma_l < 0$) to their minimum. The minimal value of the population size $D_{\min}(t_p, \omega)$ is similarly attained when the system moves in the direction of decreasing the values of

stimulators $g_{lp}(\omega, k) = \widehat{L}_{lp}(\omega, k)$ to their minimum (when $\gamma_l > 0$) and increasing the values of the destimulators of the social system $g_{lp}(\omega, k) = \widetilde{L}_{lp}(\omega, k)$ to their maximum (when $\gamma_l < 0$). It is obvious that the population size assumes a value that minimizes the functional J_1 when $J_2 = 0$, within the internal domain of the set \widetilde{G} when $D_{\min}(t_p, \omega) < D_r(t_p, \omega) < D_{\max}(t_p, \omega)$, or on the boundary of the set \widetilde{G} when $D_r(t_p, \omega) > D_{\max}(t_p, \omega)$ or $D_r(t_p, \omega) < D_{\min}(t_p, \omega)$. In the latter case, optimal values of control parameters are located on the boundary of the domain $g_{lp}^*(\omega, k) = \widehat{L}_{lp}(\omega, k)$ (when $\gamma_l > 0$), $g_{lp}^*(\omega, k) = \widetilde{L}_{lp}(\omega, k)$ (when $\gamma_l < 0$) or $g_{lp}^*(\omega, k) = \widetilde{L}_{lp}(\omega, k)$ (when $\gamma_l > 0$), and $g_{lp}^*(\omega, k) = \widehat{L}_{lp}(\omega, k)$ (when $\gamma_l < 0$). It is also obvious that an optimal control within the internal domain \widetilde{G} can be not unique. Then a unique solution is chosen by means of an additional requirement of the minimization of the computational complexity of the algorithm, for example, by the construction of a minimizing sequence by formulas (9) and (10).

5. Compute the value (s) of the control parameter

$$g_{lp}^{(s)} = g_{lp}^{(s-1)} + \Pi(\alpha^{(s)}(\omega, k)\Delta_{lp}(\omega, k)), \quad g_{lp}^{(0)} = 0, \quad \alpha^{(1)}(\omega, k) = 1 \quad (s \geq 1), \quad (10)$$

where $0 < \alpha^{(s)}(\omega, k) < 1$ is an auxiliary scalar parameter and Π is the operator of projection onto the boundary of a domain $g_{lp}^{(s)}(\omega, k) \in [\widetilde{L}_{lp}(\omega, k), \widehat{L}_{lp}(\omega, k)]$. Compute the coordinates of the (s)-element of the minimizing sequence, i.e., the system trajectory

$$x_l^{(s)}(t_p, \omega, k, g_{lp}) = \tau_0 g_{lp}^{(s)}(\omega, k) + x_l(t_{p-1}, \omega, k, g_{lp}), \quad \omega \in \Omega_1, \quad l = \overline{1, n} \quad (s \geq 1). \quad (11)$$

Compute the values of functions $u(t_p, x^{(s)}, \omega, k)$, $M(u(t_p, x^{(s)}, \omega, k, g))$, and $D(t_p, x^{(s)}, \omega, k, g)$.

6. Check the following convergence condition with the help of the minimum residual condition (the proximity of components of the gradient of the functional to zero) with a given error ε_1 :

$$(D_r(t_p, \omega, k) - D(t_p, x^{(s)}, \omega, k, g^{(s)}))^2 \leq \varepsilon_1 \|\widetilde{D}(T_0)\|_{C(\Omega \times K_0)}^2. \quad (12)$$

If condition (12) is true, then go to item 7. Otherwise, check the following conditions of the monotony of the residual:

$$(D_r(t_p, \omega, k) - D(t_p, x^{(s)}, \omega, k, g^{(s)}))^2 < (D_r(t_p, \omega, k) - D(t_p, x^{(s-1)}, \omega, k, g^{(s-1)}))^2, \quad (13)$$

$$(D_r(t_p, \omega, k) - D(t_p, x^{(s)}, \omega, k, g^{(s)})) \times (D_r(t_p, \omega, k) - D(t_p, x^{(s-1)}, \omega, k, g^{(s-1)})) > 0. \quad (13a)$$

If conditions (13) and (13a) are true, increase s by unity, assume that $\alpha^{(s)} = \alpha^{(s-1)}$, and go to the beginning of item 5 to compute the system trajectory at the new step of iterations. Otherwise, decrease the value of $\alpha^{(s)}$ by a factor of two and check the following condition of attainability of the boundary of resources in the region: $\alpha^{(s)} < \varepsilon_2$. If this condition is true, then go to item 7 and, otherwise, go to the beginning of item 5 and compute the (s)-values of the control parameter and system trajectory once again.

7. If the set Ω_1 is not exhausted, then go to item 3 and, otherwise, go to item 8.

8. Check the following condition of violation of the migration balance as a result of a change in control parameters in regions $\omega \in \Omega_1$:

$$\Delta M_{\Omega_1} = \sum_{\omega \in \Omega_1} M(t_p, x, \omega, k, g) - \sum_{\omega \in \Omega_1} M(t_{p-1}, x, \omega, k, g) = 0. \quad (14)$$

If condition (14) is true, then the migratory balance is preserved in regions $\omega \in \Omega_1$ and this implies the preservation of the balance in regions $\omega \in \Omega_2$. Since the condition of absence of risk is true at the moment t_{p-1} , put the values of $g_{lp}^{(s)}(\omega, k) = 0$ for control parameters for regions $\omega \in \Omega_2$ and go to item 11.

Otherwise, for each region $\omega \in \Omega_2$, determine maximally admissible values of control parameters as follows:

$$g_{lp}(\omega, k) = \begin{cases} \widehat{L}_{lp}(\omega, k) & \text{if } (\gamma_l(k)\Delta M_{\Omega_1}) < 0, \\ \widetilde{L}_{lp}(\omega, k) & \text{if } (\gamma_l(k)\Delta M_{\Omega_1}) > 0. \end{cases} \quad (15)$$

Since the migration balance equation has been true at the previous moment of time t_{p-1} , based on the analysis of the change in the direction of the migration process (obtained as a result of solution of the optimization problem for the domain Ω_1), the greatest possible social and economic resources of the domain Ω_2 at the current step are determined that are necessary for the compensation of the arising disbalance. For greatest possible controls (15), compute the system trajectory $x_l^l(t_p, \omega, k, g_{lp})$ by formula (11) and the values of functions $u(t_p, x^{(l)}, \omega, k)$, $M(u(t_p, x^{(l)}, \omega, k, g))$, and $D(t_p, x^{(l)}, \omega, k, g)$. Taking into account risk, correct the value of the population size resource as follows:

$$D_r(t_p, x^{(l)}, \omega, k, g) = \begin{cases} D(t_p, x^{(l)}, \omega, k, g) & \text{if } D(t_p, x^{(l)}, \omega, k, g) \in [\tilde{D}(t_p, \omega, k), \hat{D}(t_p, \omega, k)], \\ \hat{D}(t_p, \omega, k) & \text{if } D(t_p, x^{(l)}, \omega, k, g) > \hat{D}(t_p, \omega, k), \\ \tilde{D}(t_p, \omega, k) & \text{if } D(t_p, x^{(l)}, \omega, k, g) < \tilde{D}(t_p, \omega, k). \end{cases} \quad (16)$$

Determine the corresponding values of M_r by the formula

$$M_r(t_p, x^{(l)}, \omega, k, g) = \frac{1}{\tau_0} (D_r(t_p, x^{(l)}, \omega, k, g) - D(t_{p-1}, x, \omega, k, g)) - (M_E(t_p, \omega, k) + R(t_p, \omega, k)).$$

By virtue of expressions (15) and (16), the following inequality is true:

$$\sum_{\omega \in \Omega_2} (M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) - \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \leq 0. \quad (17)$$

9. If we have

$$\left| \sum_{\omega \in \Omega_2} (M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right| < \left| \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right|$$

or

$$\sum_{\omega \in \Omega_2} (M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) = 0,$$

then the socioeconomic resources in regions $\omega \in \Omega_2$ are insufficient (if any) for the solution of the demographic problem in the domain Ω_1 (under the condition that risk is absent). The problem is reduced to the problem of distribution of the available resources from regions $\omega \in \Omega_2$ among regions $\omega \in \Omega_1$. In this case, the situation of redistribution of the demographic resources between regions $\omega \in \Omega_1$ can arise. Therefore, we take a subset of regions Ω_1^+ ($\Omega_1 = \Omega_1^- \cup \Omega_1^+$, $\Omega_1^- \cap \Omega_1^+ = \emptyset$) that can be demographic “donors” for the subset of regions Ω_1^- from the ordered set of $\omega \in \Omega_1$. Then, for all $\omega^* \in \Omega_1^+$, we have

$$M(t_p, x^{(s)}, \omega^*, k, g) - \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \leq 0.$$

Using the procedures of items 4–6, determine the values of control parameters for which the trajectories at a moment of time t_p take values in the set $[x_l(t_{p-1}, \omega, k, g_{lp}), x_l^{(s)}(t_p, \omega, k, g_{lp})]$, $\omega \in \Omega_1^-$, $l = \overline{1, n}$, that minimize the residuals

$$\left(D_r(t_p, \omega, k) - D(t_p, x, \omega, k, g) \right)^2 \leq \varepsilon_1 \|\tilde{D}(T_0)\|_C^2(\Omega \times K_0)$$

and, at the same time, satisfy the migration balance equation

$$\sum_{\omega \in \Omega} M(u(t, x, \omega, k, g)) = 0.$$

In regions $\omega \in \Omega_1^+$, the values of control parameters remain unchanged. For regions $\omega \in \Omega_2$, the set \tilde{G} in problem (22) consists of one collection of maximally admissible control parameters $\{g_p^{(l)}\}$ (the entire resource of regions is used for the

solution of optimization problem (21)). This collection is assumed to be optimal. Go to item 11.

10. If we have

$$\begin{aligned} & \left| \sum_{\omega \in \Omega_2} (M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right| \\ & \geq \left| \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right|, \\ & \sum_{\omega \in \Omega_2} (M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \neq 0, \end{aligned}$$

then regions $\omega \in \Omega_2$ contain sufficient socioeconomic resources for the solution of demographic problems in regions $\omega \in \Omega_1$. Based on the migration balance equation at moments of times t_{p-1} and t_p , write an additional condition of equiproportional load connected with extraction of demographic resources in each region $\omega \in \Omega_2$. Search for the migration balance in regions $\omega \in \Omega_2$ in the form

$$M(t_p, \omega, k) = M(t_{p-1}, \omega, k) + \beta (M_r(t_p, \omega, k) - M(t_{p-1}, \omega, k)),$$

where β is a scalar parameter determined from the migration balance equation,

$$\beta = - \frac{\sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k))}{\sum_{\omega \in \Omega_2} (M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k))}.$$

By virtue of condition (17), we have $0 < \beta \leq 1$. The migration balance and the corresponding population size for regions $\omega \in \Omega_2$ are as follows:

$$\begin{aligned} M(t_p, \omega, k) &= M(t_{p-1}, \omega, k) - \frac{M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)}{\sum_{\omega \in \Omega_2} (M_r(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k))} \Delta M_{\Omega_1}, \\ D_r(t_p, \omega, k) &= D(M(t_p, \omega, k)). \end{aligned}$$

Using the procedures of items 4–6, determine the values of control parameters that minimize the residuals for regions $\omega \in \Omega_2$,

$$\left(D_r(t_p, \omega, k) - D(t_p, x, \omega, k, g) \right)^2 \leq \varepsilon_1 \|\tilde{D}(T_0)\|_C^2(\Omega \times K_0)$$

and go to item 11.

11. Check the condition of proximity of the value of the functional J_1 to zero with a prescribed accuracy or the attainment of a final moment of time,

$$J_1(t_p, k, g) = \sum_{\omega \in \Omega_1} \lambda_1(\omega) [\tilde{D}(T_0, \omega, k) - D(t_p, x, \omega, k, g)]^2 \leq \varepsilon_1 \|\tilde{D}(T_0)\|_C^2(\Omega \times K_0) \quad \text{or } p = P_0. \quad (18)$$

If condition (18) is true, then output the result and go to item 12; otherwise, go to item 2.

12. Repeat this algorithm for all the social groups $k \in K_0$.

SCENARIO OF OPTIMAL CONTROL WITH RISK MINIMIZATION

For problem (1), we consider the case when $A = E$ is a unit operator. Additional constraints on the scenario are formulated in the form

$$\tilde{g} = \text{Arg min}_{g \in G} (J_1(T_0, k, g)), \quad \tilde{g} \in \tilde{G}, \quad (19)$$

$$g^* = \text{Arg min}_{g \in \tilde{G}} (\tilde{J}_2(T_0, k, g)), \quad g^* \in G^*,$$

$$\tilde{J}_2(T_0, k, g) = \sum_{\omega \in \Omega \setminus \Omega_1} \lambda_2(\omega) r(T_0, \omega, k, g). \quad (20)$$

Condition (19) requires the minimization of deviations of the population size from the values specified by the Center on the territory Ω_1 , and restriction (20) requires the risk minimization in the other part of the territory of the domain Ω_2 for admissible optimal controls obtained in Ω_1 . Taking into account the nonnegativity of the functionals J_1 and J_2 , conditions (19) and (20) mean that the Center C_0 considers the scenario of development of the migration process when, in problem (1), the desirable distribution of social groups among given regions of the domain Ω_1 is more preferable (the functional J_1) than the absence of demographic risk in this territory (the functional J_2), i.e., the weight coefficient $\lambda_1(\omega) \gg \lambda_2(\omega)$, $\omega \in \Omega_1$. One should also take into account that we have $r_0(\omega, k) \geq 0$. Since controls are considered over the class of piecewise-constant bounded functions (the compactness of the set G) [1] and the operator A is continuous in R^n (1), condition (19) determines a compact set of optimal controls $g \in \tilde{G} \subset G$. Problem (1), (19), (20) is correctly stated. Speed problem (1), (19), (20) can also be formulated in terms of discrete optimization theory [9]. Considering the functional of population size (5), in each segment $[t_{p-1}, t_p]$ ($p=1, P_0$), we determine a collection of control parameters $g \in \tilde{G} \subset G$ that provide the minimum of the following functionals:

$$\tilde{g}_p(\omega, k) = \left(\tilde{g}_{1p}(\omega, k), \dots, \tilde{g}_{np}(\omega, k) \right) = \arg \inf_{g_p \in G} (J_1(t_p, k, g_p)), \quad (21)$$

$$g_p^*(\omega, k) = \left(g_{1p}^*(\omega, k), \dots, g_{np}^*(\omega, k) \right) = \arg \inf_{g_p \in \tilde{G}} (\tilde{J}_2(t_p, k, g_p)) \quad (22)$$

sequentially for each $p=1, 2, \dots$. Computations come to an end after the attainment of the minimum of functionals (21) and (22) with a preassigned error $\varepsilon > 0$ when $p=p^*$ or after the attainment of a final moment of time when $p=P_0$. The gradient of the functional J_1 for the domain Ω_1 is determined by formula (7). The gradient of the functional J_2 for Ω_2 is similarly determined by formula (8) in the fixed segment $[t_{p-1}, t_p]$ as follows:

$$\frac{\partial J_2(t_p, k, g,)}{\partial g_{l^*p}} = \sum_{\omega \in \Omega \setminus \Omega_1} \frac{2\lambda_2(\omega)}{(t_p - t_0)} \int_{t_{p-1}}^{t_p} \frac{\partial D(\tau, x, \omega, k, g)}{\partial g_{l^*p}} R(\tau, x, \omega, k, g) d\tau, \quad (23)$$

$$R(\tau, x, \omega, k, g) = (D(M(\tau, x, \omega, k, g)) - \tilde{D}(\tau, \omega, k)) \chi(\tilde{D}(\tau, \omega, k) - D(M(\tau, x, \omega, k, g)))$$

$$+ (D(M(\tau, \omega, k, g)) - \hat{D}(\tau, \omega, k)) \chi(D(M(\tau, \omega, k, g)) - \hat{D}(\tau, \omega, k)).$$

Formulas (23) and (8) implies that the sign of the l^* -component of the gradient of the functional J_2 is determined by the sign of $(D(M(\tau, x, \omega, k, g)) - \tilde{D}(\tau, \omega, k))$ or $(D(M(\tau, \omega, k, g)) - \hat{D}(\tau, \omega, k))$ and the sign of the parameter $\gamma_{l^*}(k)$. In fact, this means that optimal values of the l^* -component of the system trajectory $x_{l^*}(t_p)$ are in the segment connecting the initial value of the system trajectory with the projection of a given point onto the boundary of the compact domain determined by the constraints of problem (1). The properties of the gradients of the functionals $J_1(t_p, k, g)$ and $J_2(t_p, k, g)$ and the property of smoothness of the function of the population size $D(t_p, x, \omega, k, g)$ as a function of control parameters imply a numerical algorithm of solution of problem (1), (19), (20). We fix $k \in K_0$ (the C_k th social group).

1. Specify the initial parameters of the problem being solved and characteristic constants of problem (1).
2. Specify the current step with respect to time $p \geq 1$ and determine the moment of time $t_p = t_{p-1} + \tau_0$.
3. Fix the sequent region $\omega \in \Omega_1$. By formulas (7) and (8), determine the initial value of the auxiliary control parameter $\Delta_{lp}(\omega, k)$, $l=1, n$, as follows:

$$\Delta_{lp}(\omega, k) = \begin{cases} \widehat{L}_{lp}(\omega, k) / N_\gamma & \text{if } \left(\gamma_l(k)(D(t_{p-1}, x, \omega, k, g) - \widetilde{D}(t_p, \omega, k)) \right) < 0, \\ \check{L}_{lp}(\omega, k) / N_\gamma & \text{if } \left(\gamma_l(k)(D(t_{p-1}, x, \omega, k, g) - \widetilde{D}(t_p, \omega, k)) \right) > 0, \\ 0 & \text{if } \left(\gamma_l(k)(D(t_{p-1}, x, \omega, k, g) - \widetilde{D}(t_p, \omega, k)) \right) = 0. \end{cases} \quad (24)$$

Here, N_γ is the initial value of the number of steps along the segment connecting the initial point of the system trajectory with the boundary point determined by the boundary value of a compact set $G \subset R^n$. It follows from formulas (5), (7), and (8) that the maximal value of the population size $D_{\max}(t_p, \omega)$ is attained during the motion of the system in the direction of increasing the values of stimulators $g_{lp}(\omega, k) = \widehat{L}_{lp}(\omega, k)$ (when $\gamma_l > 0$) to their maximum and decreasing the values of destimulators of the social system $g_{lp}(\omega, k) = \check{L}_{lp}(\omega, k)$ (when $\gamma_l < 0$) to their minimum. The minimal value of the population size $D_{\min}(t_p, \omega)$ is similarly attained but in the other direction. It is obvious that the population size assumes the value that minimizes the functional J_1 in the internal domain of the set \widetilde{G} if we have $D_{\min}(t_p, \omega) < \widetilde{D}(t_p, \omega) < D_{\max}(t_p, \omega)$ or on the boundary of the set \widetilde{G} if we have $\widetilde{D}(t_p, \omega) > D_{\max}(t_p, \omega)$ or $\widetilde{D}(t_p, \omega) < D_{\min}(t_p, \omega)$. In the latter case, optimal values of control parameters are on the boundary of the domain: $g_{lp}^*(\omega, k) = \widehat{L}_{lp}(\omega, k)$ (when $\gamma_l > 0$), $g_{lp}^*(\omega, k) = \check{L}_{lp}(\omega, k)$ (when $\gamma_l < 0$) or $g_{lp}^*(\omega, k) = \check{L}_{lp}(\omega, k)$ (when $\gamma_l > 0$), and $g_{lp}^*(\omega, k) = \widehat{L}_{lp}(\omega, k)$ (when $\gamma_l < 0$). It is also obvious that the optimal control in the internal domain \widetilde{G} can be not unique. Then the unique solution is chosen with the help of additional requirements on the minimization of the computational complexity of the algorithm, for example, by the construction of a minimizing sequence by formulas (24) and (10).

4. By formulas (9) and (10), compute the values (s) of the control parameter $g_{lp}^{(s)}(\omega, k)$ and the coordinates of the (s)-element of the minimizing sequence, i.e., the system trajectory $x_l^{(s)}(t_p, \omega, k, g_{lp})$, $\omega \in \Omega_1$, $l = \overline{1, n}$ ($s \geq 1$). Compute the values of the functions $u(t_p, x^{(s)}, \omega, k)$, $M(u(t_p, x^{(s)}, \omega, k, g))$, and $D(t_p, x^{(s)}, \omega, k, g)$.

5. Check the condition of minimum of residual (12) with a prescribed error ε_1 . If condition (12) is true, then go to item 6.

Otherwise, check the conditions of monotony of residual (13), (13a). If conditions (13), (13a) are true, then increase s by unity, assume that $\alpha^{(s)} = \alpha^{(s-1)}$, and go to the beginning of item 4 to compute the system trajectory at the new step of iterations. Otherwise, reduce the value of $\alpha^{(s)}$ by a factor of two, check the condition $\alpha^{(s)} < \varepsilon_2$ of attainability of the limit of resources in this region. If this condition is true, go to item 6 and, otherwise, go to the beginning of item 4 and again compute the (s)-values of the control parameter and system trajectory.

6. If the set Ω_1 is not exhausted, then go to item 3 and, otherwise, go to item 7.

7. For each region $\omega \in \Omega_2$, determine maximally admissible values of control parameters $g_{lp}^{(l)}(\omega, k)$ by formulas (14) and (15). Since the migration balance equation has been true at the previous moment of time t_{p-1} , at the current step, based on the analysis of the change in the direction of the migration process (obtained as result of solution of the optimization problem for the domain Ω_1), determine the greatest possible socioeconomic resources of the domain Ω_2 that are necessary for compensation of the arising disbalance. For greatest possible controls (14) and (15), compute the system trajectory $x_l^{(l)}(t_p, \omega, k, g_{lp})$ by formula (11) and the values of the functions $u(t_p, x^{(l)}, \omega, k)$, $M(U(t_p, x^{(l)}, \omega, k, g))$, and $D(t_p, x^{(l)}, \omega, k, g)$. By virtue of formula (15), the following inequality is true:

$$\sum_{\omega \in \Omega_2} (M(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \times \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \leq 0. \quad (25)$$

8. If the following inequality is true:

$$\left| \sum_{\omega \in \Omega_2} (M(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right| < \left| \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right|,$$

then the socioeconomic resources in the regions $\omega \in \Omega_2$ are insufficient (if any) for the solution of the demographic problem in the domain Ω_1 . The problem is similar to that of item 9 from the previous scenario. Take a subset of regions Ω_1^+ ($\Omega_1 = \Omega_1^- \cup \Omega_1^+$, $\Omega_1^- \cap \Omega_1^+ = \emptyset$) that can be “donors” for the subset of regions Ω_1^- from the ordered set of $\omega \in \Omega_1$, i.e., for all $\omega^* \in \Omega_1^+$, the following inequality is true:

$$M(t_p, x^{(s)}, \omega^*, k, g) \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \leq 0.$$

Using the procedures of items 4–6, determine the values of control parameters for which the trajectories at the moment of time t_p assume values from the set $[x_l(t_{p-1}, \omega, k, g_{lp}), x_l^{(s)}(t_p, \omega, k, g_{lp})]$, $\omega \in \Omega_1^-$, $l = \overline{1, n}$, that minimize the residuals

$$(D_r(t_p, \omega, k) - D(t_p, x, \omega, k, g))^2 \leq \varepsilon_1 \|\tilde{D}(T_0)\|_{C(\Omega \times K_0)}^2$$

and simultaneously satisfy the migration balance equation

$$\sum_{\omega \in \Omega} M(u(t, x, \omega, k, g)) = 0.$$

The values of control parameters in regions $\omega \in \Omega_1^+$ remain unchanged. For regions $\omega \in \Omega_2$ in problem (22), the set \tilde{G} consists of one collection of maximally admissible control parameters $\{g_p\}$ (all the resources of these regions are spent for the solution of optimization problem (21)). It is this collection that is assumed to be optimal. Go to item 10.

9. If the following inequality is true:

$$\left| \sum_{\omega \in \Omega_2} (M(t_p, x^{(l)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right| \geq \left| \sum_{\omega \in \Omega_1} (M(t_p, x^{(s)}, \omega, k, g) - M(t_{p-1}, \omega, k)) \right|,$$

then regions $\omega \in \Omega_2$ contain sufficient socioeconomic resources for the solution of demographic problems in regions $\omega \in \Omega_1$. Using the procedures of items 4–6, determine the values of control parameters for which the trajectories at the moment of time t_p assume values from the set $[x_l(t_{p-1}, \omega, k, g_{lp}), x_l^{(l)}(t_p, \omega, k, g_{lp})]$, $\omega \in \Omega_2$, $l = \overline{1, n}$, that minimize the risk

$$R^2(\tau, x, \omega, k, g) = ((D(M(\tau, x, \omega, k, g)) - \tilde{D}(\tau, \omega, k))\chi(\tilde{D}(\tau, \omega, k) - D(M(\tau, x, \omega, k, g))) + (D(M(\tau, x, \omega, k, g)) - \tilde{D}(\tau, \omega, k))\chi(D(M(\tau, x, \omega, k, g)) - \tilde{D}(\tau, \omega, k)))^2 \leq \varepsilon_1 \|\tilde{D}(T_0)\|_{C(\Omega \times K_0)}^2$$

and simultaneously satisfy the migration balance equation

$$\sum_{\omega \in \Omega} M(u(\tau, x, \omega, k, g)) = 0.$$

10. Check the following condition of proximity of the functional J_1 to zero with a prescribed accuracy or the attainment of the final moment of time:

$$J_1(t_p, k, g) = \sum_{\omega \in \Omega_1} \lambda_1(\omega) \left[\tilde{D}(T_0, \omega, k) - D(\tau_p, x, \omega, k, g) \right]^2 \leq \varepsilon_1 \|\tilde{D}(T_0)\|_{C(\Omega \times K_0)}^2 \quad \text{or } p = P_0. \quad (26)$$

If condition (26) is true, then output the result and go to item 11; otherwise, go to item 2.

11. Repeat this algorithm for all social groups $k \in K_0$.

TABLE 1. Optimal Model Data on the Population Size in the Oblasts of Ukraine according to the Scenario of the Transregional Migration with Zero Risk

Oblast	D_0	\tilde{D}	\hat{D}	D_{opt}	\tilde{D}
Ω_1					
Volynsk oblast	1061.1	900	1200	1082.27	1090
Chernovtsy oblast	938.2	800	1050	956.40	970
Ω_2					
Vinnitsa oblast	1819.7	1700	1900	1803.25	–
Dnepropetrovsk oblast	3733.6	3600	3800	3701.81	–
Donetsk oblast	4987.3	4800	5100	4934.77	–
Zhitomir oblast	1437.7	1300	1500	1425.16	–
Zakarpate oblast	1280.7	1200	1350	1277.84	–
Zaporozhye oblast	2016.0	1900	2100	1999.47	–
Ivano-Frankovsk oblast	1450.3	1300	1550	1445.69	–
Kiev oblast	1844.0	1700	2000	1824.00	–
Kirovograd oblast	1172.5	1050	1300	1157.54	–
Lugansk oblast	2668.1	2500	2800	2635.05	–
Lvov oblast	2708.3	2600	2850	2696.80	–
Nikolaev oblast	1307.6	1200	1400	1297.48	–
Odessa oblast	2510.2	2400	2650	2492.57	–
Poltava oblast	1688.8	1550	1800	1672.10	–
Rovno oblast	1184.9	1050	1300	1183.16	–
Sumy oblast	1349.3	1250	1450	1333.47	–
Ternopol oblast	1159.0	1050	1300	1152.46	–
Kharkov oblast	2977.6	2800	3100	2950.80	–
Kherson oblast	1234.1	1100	1350	1225.13	–
Khmelnyskii oblast	1468.4	1350	1600	1455.51	–
Cherkassy oblast	1459.5	1350	1600	1446.76	–
Chernigov oblast	1299.8	1200	1400	1281.90	–
Kiev	2596.0	2400	2800	2602.36	–
Sevastopol	391.9	320	450	389.69	–
Autonomous Republic Crimea	2106.3	2000	2150	2094.37	–

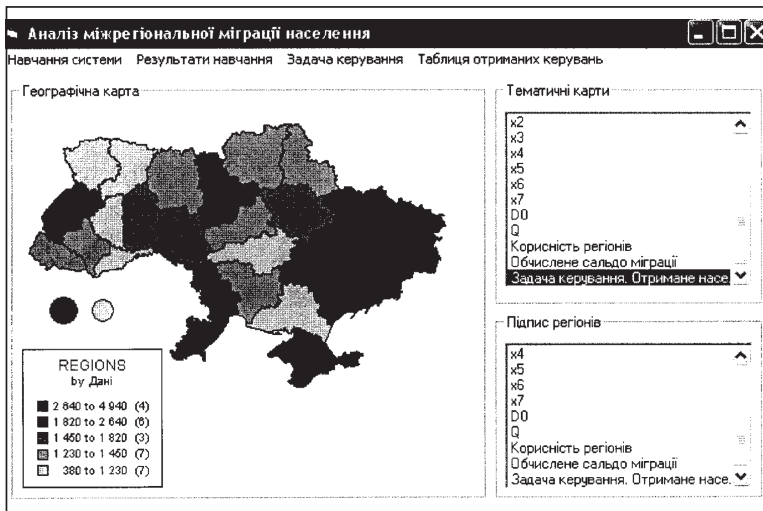


Fig. 1

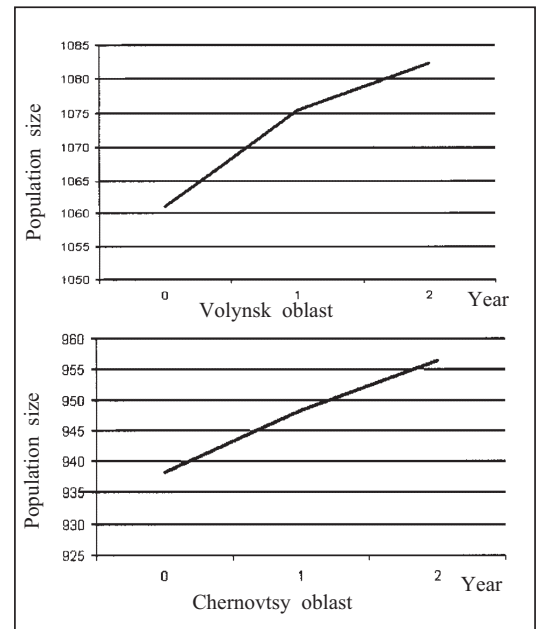


Fig. 2

NUMERICAL REALIZATION OF SCENARIOS

Based on the developed models and algorithms, an applied geoinformation system (GIS) is created that supports making decisions on the management of transregional migration processes on the territory of Ukraine [10–12]. For problem (1), the territory of Ukraine with 25 oblasts and two special centers of population (Kiev and Sevastopol), $\Omega = 27$, and one social group of “statistical average citizens,” $K_0 = 1$, are considered. The basic control parameters $j_0 = 7$ and the parameter $q = 3$ were used. Calculations were based on data from [13, 14] for Ukraine over 1999–2004.

The results of calculations for the scenarios of optimal control of transregional migration with zero risk at a final moment of time are presented in Table 1. Figure 1 presents the map of the population size in the Ukrainian regions after the solution of the control problem according to the scenario with zero risk, and Fig. 2 presents the diagrams of changing the population size in the regions of Ω_1 for the first scenario (according to the results of functioning of the GIS).

For the considered variant of problem (1), the number of variables used for the solution of extremal problems equals $N_{\text{tot}} = \Omega \times j_0 = 189$ for each fixed time interval. Therefore, the use of the developed algorithms of directed search for optimal solutions allowed us to considerably decrease the time of numerical solution in comparison with algorithms of exhaustive search over the global set and to solve this problem in real time.

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