

positions, with preference still given to men. It has been proven that a higher level of gender balance correlates with the efficiency of innovative projects, resulting in a 12–15% increase in labor productivity.

The positive factors encouraging women's participation in innovative economic recovery projects in Ukraine include:

- the opening of vacancies due to the mobilization of men;
- state strategies for overcoming the gender gap; [1]
- retraining programs for women and grant support. [2]

The results of the comparative index analysis indicate that Ukraine lags behind EU countries in terms of segregation (65.6 points versus 73.8), but demonstrates higher potential due to the high educational level of Ukrainian women.

Conclusions. The implementation of innovative projects for the economic recovery of Ukraine contributes to increasing women's employment (an 8–12 percentage point rise in the share of women), reducing the gender pay gap between men and women, and boosting women's entrepreneurship activity.

At the same time, significant gender disparities persist in managerial and technical positions across many sectors of the economy.

It should be emphasized that applying a gender-sensitive approach to the statistical analysis of changes in employment structure within Ukraine's innovative economic recovery projects enables the development of effective mechanisms to improve the employment rate and labor productivity of Ukrainian women, especially under the conditions of full-scale war.

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CLASSICAL AND QUANTUM RECURRENCE QUANTIFICATION MEASURES FOR S&P 500 MARKET DYNAMICS

Financial markets are paradigmatic complex systems whose dynamics combine stochastic fluctuations with episodes of collective, deterministic-like behaviour, particularly around crises. Recurrence plots (RP) and recurrence quantification analysis (RQA), introduced by Eckmann, Kamphorst, and Ruelle [1] and later developed into a quantitative toolbox by Webber and Zbilut [2] and Marwan, Romano, Thiel, and Kurths [3], have become a standard means of revealing hidden

temporal structure in nonlinear and non-stationary time series. A recurrence plot visualizes the moments at which the phase-space trajectory of a system returns to the neighborhood of a previously visited state, and the derived RQA measures quantify the density, length, and entropy of the resulting line structures. In parallel, the rapid development of quantum machine learning has produced quantum-inspired encodings of time series: Sokol, Volf, Hejda, and Kutílek [4] recently proposed the QTS2D library, which reformulates classical time-series-to-image transformations — including recurrence plots — using amplitude encoding and quantum fidelity. The aim of the present work is to compare classical and quantum recurrence quantification measures for the dynamics of the S&P 500 stock index and to assess whether the quantum formulation offers a complementary view of market regimes.

Following the standard delay-embedding approach, the scalar series of S&P 500 returns x_i is reconstructed into an m -dimensional phase space by formula (1):

$$v_i = (x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}), \quad i = 1, \dots, N, \quad (1)$$

where m is the embedding dimension, τ the time delay, and N the number of reconstructed state vectors. The classical recurrence matrix is obtained by thresholding the pairwise Euclidean distances between these state vectors, formula (2):

$$R_{ij} = \theta(\varepsilon - \|v_i - v_j\|), \quad (2)$$

where $\theta(\cdot)$ is the Heaviside step function and ε a fixed tolerance; $R_{ij} = 1$ marks a recurrence of state i at time j .

The quantum recurrence plot replaces the metric comparison of state vectors by a comparison of quantum states. Each embedded vector v_i is mapped to a quantum state $|\varphi_i\rangle = \Phi(v_i)$ through a parameterized quantum feature map $\Phi(\cdot)$, and the similarity of two states is measured by the quantum fidelity, formula (3):

$$F_{ij} = |\langle \varphi_i | \varphi_j \rangle|^2, \quad (3)$$

from which a quantum distance and the corresponding binary recurrence matrix are defined by formula (4):

$$D_{ij} = \sqrt{1 - F_{ij}}, \quad R_{ij}^Q = \theta(\varepsilon - D_{ij}). \quad (4)$$

Because fidelity is evaluated in a high-dimensional Hilbert space, the quantum distance is sensitive to higher-order correlations between states that the Euclidean metric does not capture [4].

From either recurrence matrix the same family of RQA measures is computed. The two most basic indicators are the recurrence rate and the determinism:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^N R_{ij}, \quad (5)$$

$$DET = \frac{\sum_{l=l_{min}}^N l \cdot P(l)}{\sum_{l=1}^N l \cdot P(l)}, \quad (6)$$

where $P(l)$ is the histogram of diagonal line lengths and l_{min} the minimal line length (here $l_{min} = 2$). The recurrence rate quantifies the overall density of recurrences, whereas the determinism measures the proportion of recurrence points that form diagonal lines and thus reflects the predictability of the dynamics. The complete set of measures used in this study is summarized in Table 1.

Table 1

Recurrence quantification measures computed for the S&P 500 series

Measure	Symbol	Interpretation
Recurrence rate	RR	Overall density of recurrence points in the plot.
Determinism	DET	Fraction of recurrence points forming diagonal lines; predictability of the dynamics.
Laminarity	LAM	Fraction of recurrence points forming vertical lines; share of laminar (trapped) states.
Mean diagonal length	L	Average length of diagonal lines; mean prediction horizon.
Trapping time	TT	Average length of vertical lines; mean duration of laminar states.
Divergence	DIV	Inverse of the longest diagonal line ($1/L_{max}$); related to the largest Lyapunov exponent.
Diagonal entropy	$ENTR$	Shannon entropy of the diagonal line-length distribution; structural complexity.

Daily closing prices of the S&P 500 index (ticker ^GSPC) were retrieved for the period 1990–2026. Classical and quantum recurrence matrices were computed with embedding dimension $m = 3$, delay $\tau = 1$, and a common tolerance/threshold $\varepsilon = 0.3$. Two complementary experiments were carried out. First, both recurrence plots were constructed for a single interval covering June 2024 – June 2026 (Figure 1). Second, the recurrence rate was tracked in a sliding window of 500 trading days, advanced in steps of five days, over the entire sample (Figure 2).

Figure 1 contrasts the classical and quantum recurrence plots of the same return interval. Both diagrams share the principal large-scale structure — the bright main diagonal and the block patterns associated with persistent volatility regimes — confirming that the quantum encoding preserves the essential recurrence geometry. The quantum plot is, however, visibly sparser and more selective: many isolated recurrence points present in the classical plot are filtered out, so that only the more strongly correlated state pairs survive the fidelity threshold. This is consistent with the higher class separability reported for quantum encodings by Sokol et al. [4].

RP / QRP for 2024-06-03 to 2026-06-10 (507 points)

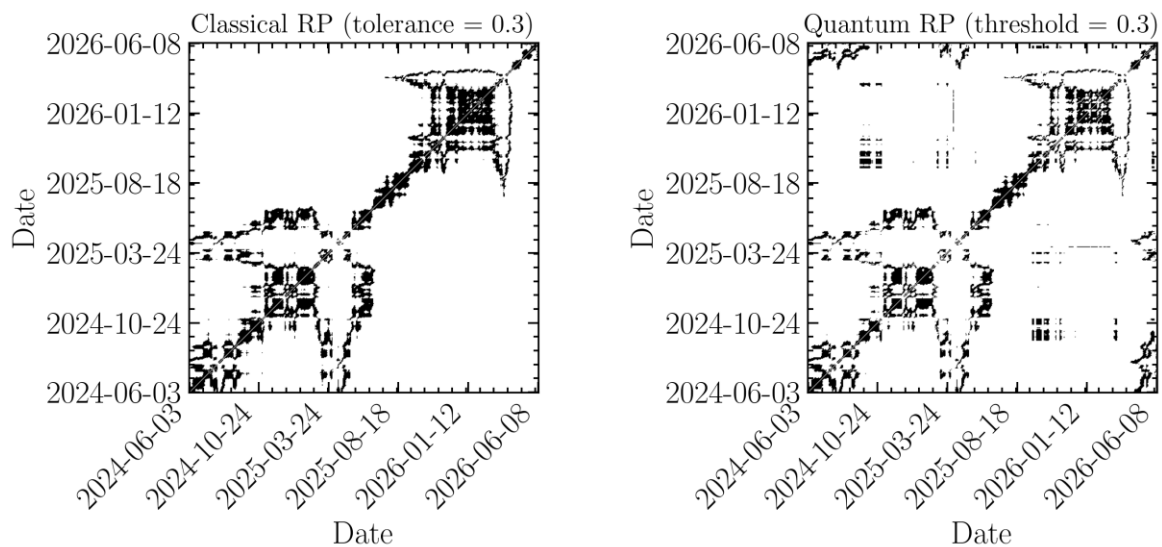


Figure 1 – Classical recurrence plot (left) and quantum recurrence plot (right) of S&P 500 returns over 2024-06-03 – 2026-06-10 (507 points) at tolerance/threshold $\varepsilon = 0.3$

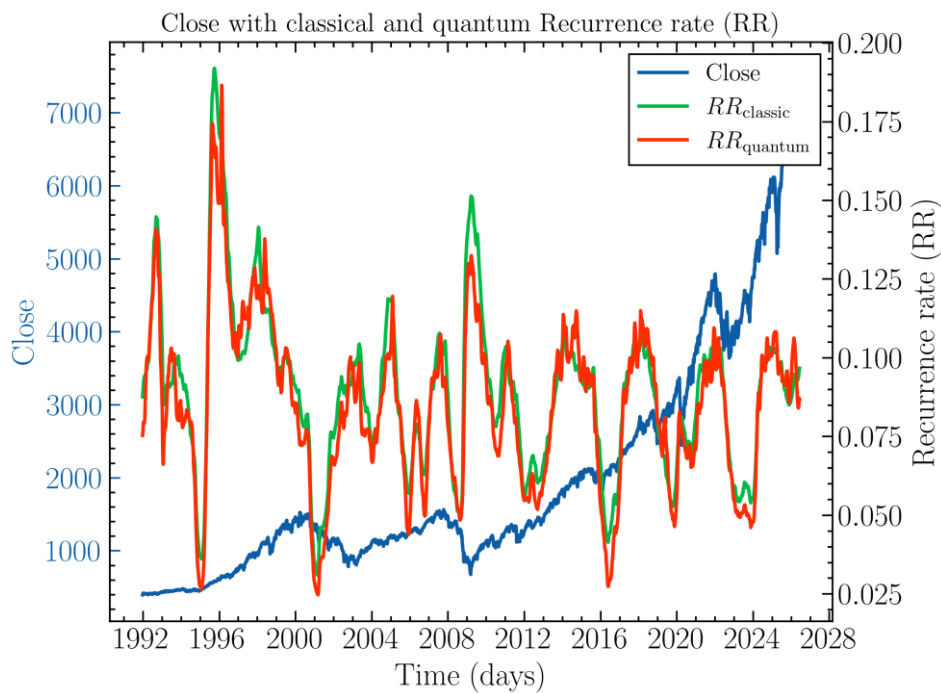


Figure 2 – S&P 500 closing price together with the classical and quantum recurrence rate computed in a sliding window of 500 trading days

Figure 2 places the classical and quantum recurrence rates alongside the index price over more than three decades. The two indicators are strongly co-monotonic and both rise sharply during well-known turbulent periods — the dot-com unwinding of 2000–2002, the 2008 global financial crisis, and the COVID-19 shock of early 2020 — when returns become more repetitive and the market visits a narrower set of states. The quantum recurrence rate follows its classical counterpart closely but displays sharper, more pronounced peaks at these turning points, suggesting that the fidelity-based measure is somewhat more responsive to abrupt regime changes.

The results show that classical and quantum recurrence quantification measures provide largely consistent but not identical descriptions of S&P 500 dynamics. The quantum recurrence plot retains the dominant recurrence structure while acting as a stricter filter, and the quantum recurrence rate behaves as a slightly more sensitive crisis indicator. Quantum-inspired recurrence measures can therefore serve as a complementary tool for detecting and characterizing market regimes [5]. Their main current limitation is computational cost, since fidelity evaluation scales quadratically with the number of states; future work will address efficient implementation and the systematic comparison of the full RQA measure set across a broader range of financial assets.

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