

the fraction of households, in which expenditures exceeded income. As it could be expected, the highest level of the said fraction concerns lowest-income households, with wealthy households with the lowest level. Unexpectedly, the lowest intensity of income insufficiency against the incurred expenditures characterises households from the middle income decile classes, and not from the expected last classes. Also, the decreasing trend was noticed in the level of expenditures for food and the increase trend in location maintenance expenditures and the fact that structure of expenditures depends on the amount of income obtained per person. The analysis of the fraction of households, which did not incur expenditures within particular expenditure sections allowed, with a dose of reservation, to define hierarchy in meeting household needs.

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Yuri Chernyavsky,

Kyiv National Economic University

STATISTICAL ESTIMATION OF THE UKRAINIAN HOUSEHOLD'S DIFFERENTIATION BY INCOME

It is important to estimate correctly the household's differentiation by the income at the present stage of the development of the Ukrainian economy. It is defined by the necessity to elaborate the effective and adequate state decisions for area of the realization social and economic actions, which must be directed on the rise of a living standard and decrease of property household's stratification.

The results of the analysis and modeling of the household's distribution by the income can be useful to tax bodies for comparison an actual and probable the taxes payment. Also, it is important at a microlevel, for instance for the formation of a policy of the production manufacture and selling by the producers of the goods and services, which are guided on different profitable household's groups.

The statistical data of sample survey of household living conditions have been chosen from existing sources of the information on household's incomes. The results of the surveys enable to carry out the distribution of the households by the material prosperity. Thus, it

were used the parameters of the expenditure, as more reliable in comparison with parameters of the incomes for an estimation of the differentiation Ukrainian households.

It is necessary to note, that the analysis of the households distribution assumes the research of two basic questions. First, it is the definition of the distribution's form and secondly, it is an estimation of the non-uniformity of household's distribution.

The researchers proved, that the incomes submit to the lognormal distribution. Thus, the parameter (x) is called the normal, if its natural logarithm ($\ln x$) submits to the lognormal distribution. Parameters of the scale and shape of the distribution Ukrainian households by monthly average total expenditure per capita for last five years are given in the following table.

Table 1

**STATISTICAL PARAMETERS OF THE SCALE AND SHAPE
OF THE DISTRIBUTION UKRAINIAN HOUSEHOLDS BY
MONTHLY AVERAGE TOTAL EXPENDITURE PER CAPITA**

Parameters of the distribution	1999	2000	2001	2002	2003
Mean, UAH	170,9	215,8	242,6	264,2	301,3
Mode, UAH	116,3	146,9	162,6	185,1	226,1
Median, UAH	149,0	186,6	210,4	230,1	261,5
Coefficient of skewness	1,869	1,494	1,400	1,343	1,127

The correlation of a mode, median, mean and coefficient of skewness point at significant right tail skewness of the household's distribution by the expenditure. Therefore subordination of the random variable to the lognormal distribution has been checked up.

Empirical and normal distribution of the households by the transformed variable (natural logarithm of the monthly average total expenditure per capita) are represented in Figure 1.

$\ln X = \ln$ (monthly average total expenditure per capita)

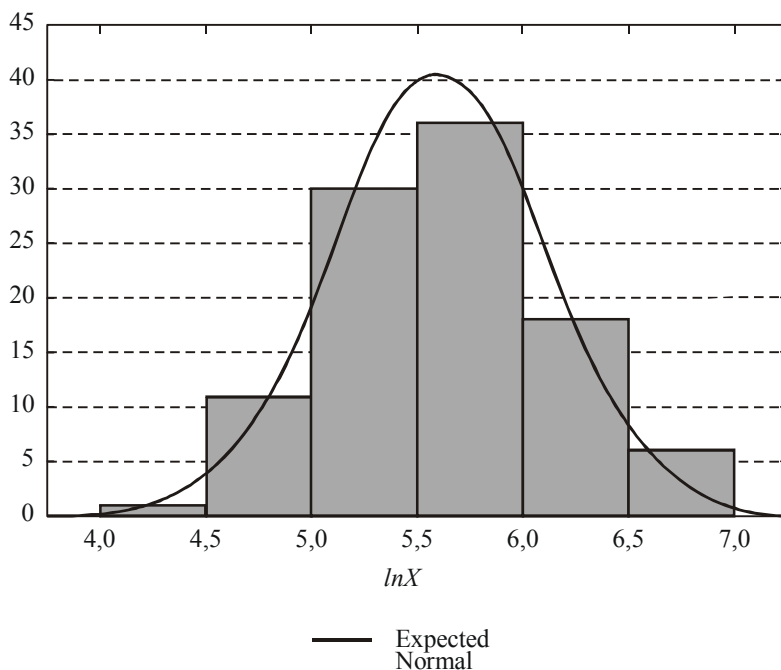


Figure 1. Empirical and expected normal distribution of the households by the transformed variable

The Kolmogorov-Smirnov test has been used for an estimation of the coordination of empirical distribution with normal distribution. Theoretical Kolmogorov-Smirnov statistic (0.14) exceeds actual statistic at $\alpha = 0,05$. Thus, the received results testify that the household's distribution by the expenditure is subordinated to the lognormal distribution.

The lognormal distribution has the probability density function:

$$p_n(x) = \frac{1}{\sqrt{2\pi\sigma x}} e^{-\frac{(\ln x - \ln a)^2}{2\sigma^2}} \quad (1)$$

where a is the scale parameter, σ is the shape parameter, e is Euler's constant.

Probability distribution function looks as follows:

$$F_\eta(x) = \frac{1}{\sqrt{2\pi\sigma}} \int_0^{\ln x} e^{-\frac{(t - \ln a)^2}{2\sigma^2}} dt. \quad (2)$$

The probability density function and probability distribution function of the lognormal distribution are represented in Figure 2.

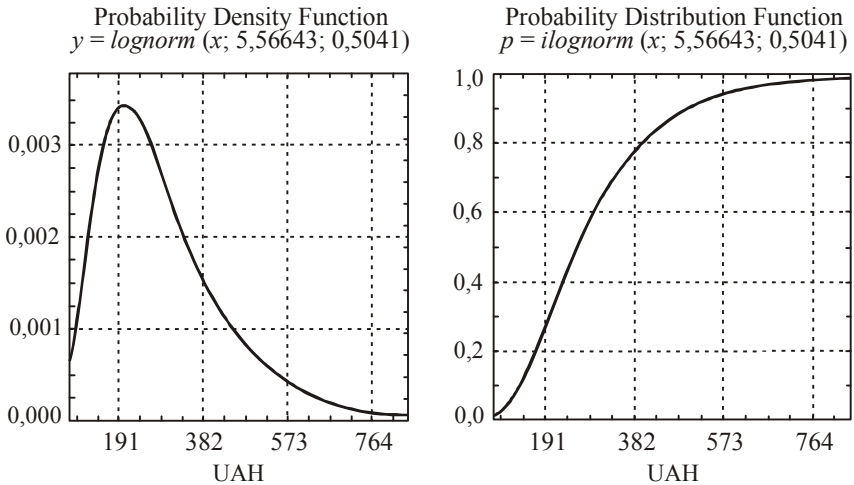


Figure 2. Lognormal distribution of the Ukrainian households by the expenditure in 2003

It is necessary to note, that reliability of the statistical data in the tail of the distributions has great significance at the calculation of the differentiation's parameters (for example, Index Gini, quantile index for differentiating etc). The understating of a level of the most prosperity household's incomes can result in to distortion of the differentiation estimations. It is obvious, that real distribution of the households is a mix of the distributions of the households, which belong to different profitable groups.

The truncated distributions are used in cases when incomes, which exceed the certain size, are of interest to researches. Pareto distribution has been used for the description of the distribution the most prosperity Ukrainian households.

Italian statist Pareto used the data of tax statistics. He has come in a conclusion, that the distribution of the prosperity household's incomes can be described to the sedate function of the following kind:

$$p_{\eta}(x) = \frac{\alpha}{c_0} \left(\frac{c_0}{x} \right)^{\alpha+1} \quad (3)$$

where α is the shape parameter of the Pareto distribution, c_0 is the mode of the distribution.

The Pareto distribution has the probability distribution function:

$$F\eta(x) = 1 - \left(\frac{c_0}{x}\right)^\alpha \quad (4)$$

The probability density function of the Pareto distribution is represented in Figure 3. The median of the lognormal distribution is accepted as a fashion of the Pareto distribution for the simplification of the calculations.

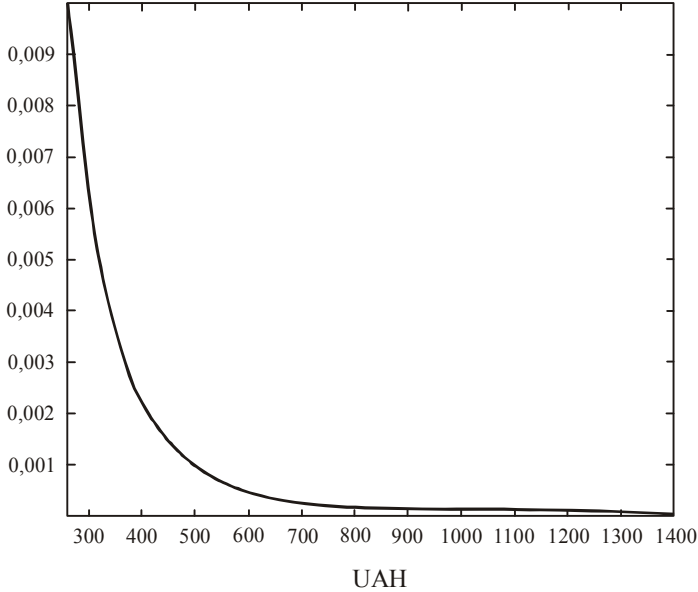


Figure 3. Pareto distribution of the most prosperity Ukrainian households by the expenditure in 2003

Thus, the model of household's distribution as a mix of the distributions is constructed on condition that 50 % of the least provided Ukrainian households are distributed according to the lognormal distribution and other half of the most provided households according to the Pareto distribution.

The opportunity to connect the probability density functions is provided according to that the density of any distribution (in this case it is density of a mix of the distributions) limits on a plane a figure with the area, which is equal to one:

$$\int_{-\infty}^{\infty} p(x) dx = 1. \quad (5)$$

Inasmuch probability density function of the lognormal distribution at $x = \text{median}$ of the lognormal distribution limits a figure with the area, then the mix of the distributions should limit a figure with the area on a plane:

$$\int_{-\infty}^{\text{med}} p(x)^{\log normal} dx + \frac{1}{2} \int_{\text{med}}^{\infty} p(x)^{\text{Pareto}} dx = 1. \quad (6)$$

Therefore the density of the Pareto distribution has to divide on two before the joining. It will have such the equation in a point of the crossing:

$$p\eta(x)^{\log normal} = \frac{1}{2} p\eta(x)^{\text{Pareto}} \quad (7)$$

Recognizing that $x = a$ (median of the lognormal distribution) and $x = c$ (mode of the Pareto distribution), it is necessary to calculate an average quadratic deviation of the natural logarithm of the monthly average total expenditure per capita for a calculation of the shape parameter of the Pareto distribution:

$$\frac{1}{\sqrt{2\pi\sigma}} = \frac{\alpha}{2}. \quad (8)$$

The curve, which consists of two parts, is represented in figure 4: It coincides with probability density function of the lognormal distribution up to a median, and it coincides with probability density function of Pareto distribution after a median.

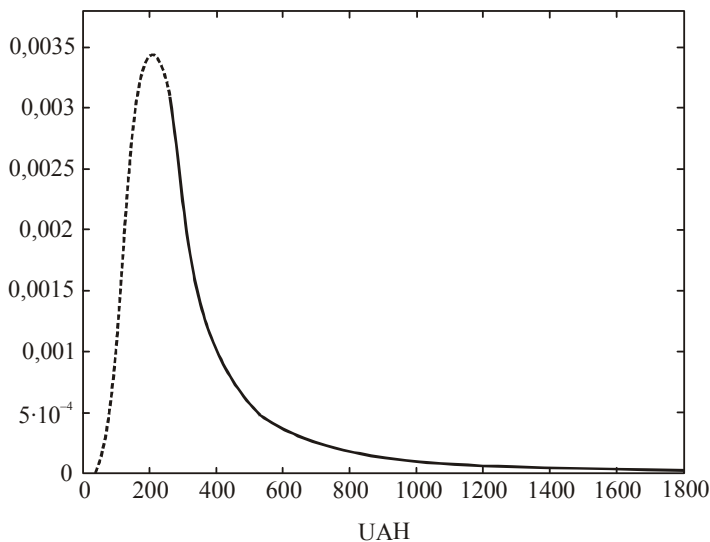


Figure 4. Distributions of the Ukrainian households by the expenditure according to the model of mix distributions in 2003

Thus, the probability distribution function this model coincides with probability distribution function of the lognormal distribution for 50 % of the least provided Ukrainian households and it coincides with probability distribution function of the Pareto distribution for other half of the most provided households:

$$F\eta(x) = \int_{-\infty}^{med} p(x)^{\log normal} dx + \frac{1}{2} \int_{med}^{\infty} p(x)^{Pareto} dx \quad (9)$$

The comparative characteristic of the parameters, which estimate the differentiation of the Ukrainian households, is given according to the empirical and the modeled distributions.

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The shares of the households, which have the monthly average total expenditure per capita under a fixed level according to the distribution, are given in the following table.

Table 2

SHARE OF HOUSEHOLDS DEPENDING ON THE MONTHLY AVERAGE TOTAL EXPENDITURE PER CAPITA, 2003

Monthly average total expenditure per capita, UAH	Share of households with the given level of the expenditures according to the distribution		
	Empirical	Lognormal	Lognormal-Pareto
Under 300	67,75	60,74	59,32
Under 450	83,34	85,92	77,86
Under 600	94,11	95,03	85,63

It is obvious, that theoretical distributions differ insignificantly up to the certain border. But with the increase a level of the expenditures, the shares of the households, which have reached a respective level of the expenditures according to the mix distribution model in comparison with the empirical and lognormal models more differ. Besides the level of the monthly average total expenditure per capita for 10 %, 5 % and 1 % of the most provided households is calculated.

The data of the table 3 testify to a significant difference between the minimal monthly average total expenditure per capita, which households should carry out so that to get in top quantiles.

Table 3

**MINIMAL MONTHLY AVERAGE TOTAL EXPENDITURE PER CAPITA
OF THE MOST WELL OFF UKRAINIAN HOUSEHOLDS, 2003**

Highest quantiles	Minimal monthly average total expenditure per capita according to the model of the distribution, UAH	
	Lognormal	Lognormal-Pareto
10 %	498,9	764,0
5 %	599,2	1212,3
1 %	844,8	3541,8

Quantile indexes for differentiating, which are calculated according to the empirical data and to the model of the lognormal distribution, differ insignificantly — accordingly 3,8 and 3,6. Minimal expenditure of the richest households 5,6 times as big in comparison with the maximum expenditure of the poorest households according to the model of the Lognormal-Pareto distribution.

According to the empirical data the index Gini equal 27,4 % in 2003. It testifies to the moderate difference of the Ukrainian households by the income. But according to the Lognormal-Pareto distribution the index Gini equal 41,1 % in 2003. It points to the significant difference of the Ukrainian households by the income.

Thus, the smoothed estimations are received according to the lognormal model. It leads to understating of the differentiation's level of the Ukrainian households.

But the model of mix distributions has some lacks, which influence accuracy and adequacy of the results. The subsequent perfection of the offered model is defined, for example a find of an optimum point of the connection the probability density function of the lognormal and Pareto distributions.

Thus, theoretical distribution of the households well describes lognormal distribution for low profitable groups, and other theoretical distributions are suitable more for the most provided households. It should provide reliability of statistical estimations of the differentiation the Ukrainian households by the income.

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Volodymyr Sarioglo,

National Academy of Science of Ukraine,
Institute for Demography and Social Research,

STATISTICAL ESTIMATION OF THE PAYMENT CAPACITY AT THE MUNICIPAL LEVEL

Scientifically proved approach to development of a tariff policy at the state, regional and local levels is the important precondition of social and economic development of regions. In market economy the tariff policy is appreciably defined by ability-to-pay of the population and the enterprises at the local level.

Problem of a possibility of the adequate analysis of the ability-to-pay of municipal services consumers gets a special urgency in process of dynamic reforming of housing-and-municipal sphere in Ukraine and in other post-soviet states [1, 2]. Reliable definition of the population ability — and willingness-to-pay for the consumed municipal services enables development and introduction of an effective social and tariff policy, overcoming of a consumer's debts problem, increase of appeal of the municipal enterprises for investors, planning of the municipal development programs, etc.

In such conditions the role of the supply with information of the governmental and legislative structures activity on the state, regional and local levels as well as administration of municipal enterprises, political and public organizations essentially increases.

The known international organizations, such as the World Bank, the European Bank of Reconstruction and Development, the US Agency on the International Development and oth. are engaged in researches of the population ability-to-pay in housing-and-municipal sphere.

Separate researches in Ukraine have been executed within 2001—2003 by PADCO advisory firm at fulfillment of work under the project «Reforming of tariffs and re-structuring of the municipal enterprises in Ukraine» [3, 4]. Now the certain works in this direction are carried out in the Institute for Demography and Social Research of