

divorce. It is essential to reduce the consequences of divorce or separation which lead to psychic deprivation of children, which in turn often results in psychological disturbances, loss of interest, activity and reduction in the degree of professional and family success.

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PROBLEMS OF MEASURING AGEING PROCESS OF POPULATION

1. Age structure of population as an illustration of ageing process

Ageing process is a consequence of demographic transition process in European countries, connected with industrialisation process and social modernisation of Euro-Atlantic civilisation³³. It also can be understood as a determinant of social maturity of population [Frączak (2002)].

Ageing process of population can be expressed by age structure (pyramid) of population. According to G. Sundbärg, three types of age structure can be distinguished: progressive, stable and regressive structure of population³⁴. Model structures of population are presented at fig. 1.

³³ More about it see: Okólski (ed) (1990).

³⁴ See for example: Cieślak (1992), Holzer (2003), Kurkiewicz (1992).

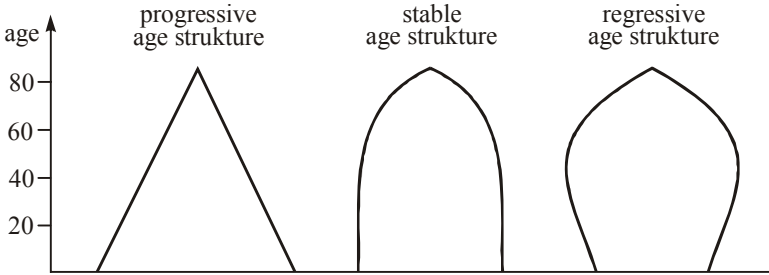


Figure 1. Model structures of population

Empirical structure of population gives us the first information about ageing process of population. When we observe progressive type of age structure, the population is young with increasing number of population. In case of stable structure, when height of pyramid is increasing, we observe ageing process of population from the top of age structure, connected with extending of life span. Regressive age structure informs us about ageing process from the bottom of age pyramid.

Let us see on age structure of Polish population at 2002, what shows fig. 2.

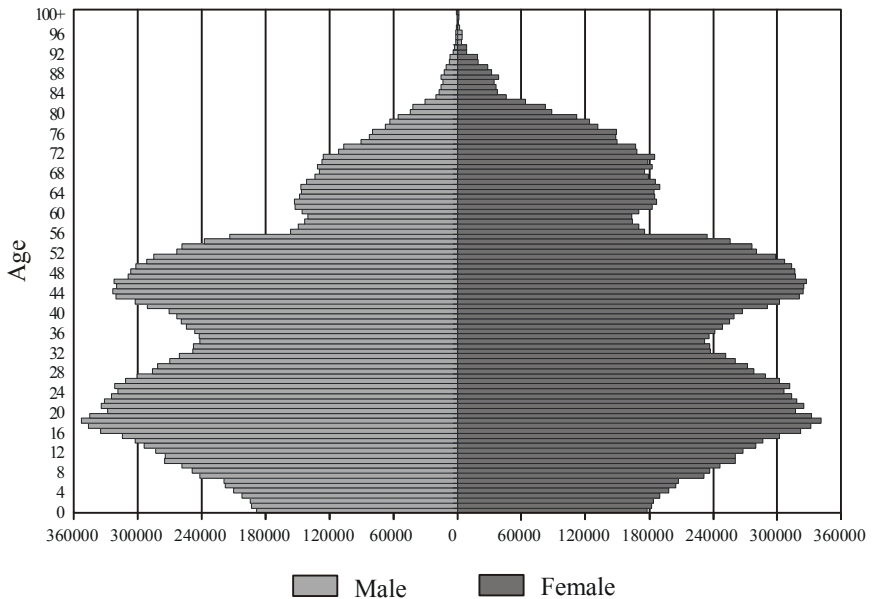


Figure 2. Polish population by age and sex in 2002

As we can see, polish population is still relatively young, but ageing process occurs from the bottom of the pyramid.

2. Indices of ageing process

The ageing process can be measured in various manners [Frątczak (2002)]. The basic way of measurement is comparison of the number of population in old age to the total number of population. There emerge the problem of determination of old age threshold. According to French demographers and Polish demographer E. Rosset [Rosset (1959)] the old age threshold is the age of 60 years. U N standards take into account 65 years as the old age threshold. Therefore the old age ration can be defined as follows:

$$W_t^{(1)} = \frac{l_{t60+}}{L_t} \cdot 100 \quad (1)$$

or:

$$W_t^{(2)} = \frac{l_{t65+}}{L_t} \cdot 100 \quad (2)$$

where:

L_t — total number of population,

l_{t60+}, l_{t65+} — number of population in age of 60 or 65 years and over.

Another possibility of measuring of ageing process is comparison of the number of population in old age to the number of young population:

$$I_t^{(1)} = \frac{l_{t60+}}{l_{t0-14}} \cdot 100 \quad (3)$$

or Sauvy's index:

$$I_t^{(2)} = \frac{l_{t60+}}{l_{t0-19}} \cdot 100 \quad (4)$$

where:

l_{0-14}, l_{0-19} — number of population in age 0 – 14 or 0 – 19 years.

Sauvy's index is interpreted as the proportion of grandparents to grandchildren.

Demographic interpretation values of indices mentioned above based on scales of old age, widely discussed in literature³⁵.

An important is dependency ratio, which determine the proportion between economically inactive and active populations:

³⁵ See for example: Kurek (2001) or Kurkiewicz (1992).

$$W_{O_t} = \frac{l_s}{l_p} \cdot 100 \quad (5)$$

where:

l_s — the number of population in retirement age (females 60+, males 65+ in Poland),

l_p — the number of population economically active (females 18 – 59, males 18 – 64 in Poland).

An interesting is also investigation the structure of population in old age. The simplest measure is advanced old age ratio:

$$W_{S_t} = \frac{l_{80+}}{l_{60+}} \cdot 100 \quad (6)$$

where:

l_{80+} — is the number of population in age 80 and more.

In investigation of ageing process is important not only how advanced is this process but also what is the dynamics of ageing process. The simplest measure of dynamics is growth rate of ageing process:

$$\delta W_t = \frac{W_t - W_{t-1}}{W_{t-1}} \quad (7)$$

Index numbers of ageing process can be also applied:

$$\text{or } i_{t/0} = \frac{W_t}{W_0} \cdot 100 \quad (8)$$

$$i_{t/t-1} = \frac{W_t}{W_{t-1}} \cdot 100 \quad (9)$$

A tendency of ageing process can be also described by trend function. We have to determine its analytical shape (linear or not linear) and possibilities of demographic interpretation. We can also analyse changes of age median as the most synthetic measure of ageing process.

3. Skewness of age distribution as a measure of ageing

Let us see on age distribution without gender distinction (on example of age structure of Polish population in 2002), what is presented on fig. 3.

Age distribution is right-side asymmetric. When the level of asymmetry becomes weaker, the ageing process occurs. When age distribution becomes symmetrical or even left side asymmetric, means that ageing process is advanced. Cieślak (2003a) and Cieślak (2003b)

proposed the measure of asymmetry as the tool of determining how ageing process is advanced. She applied Charlie's — Wywiał's asymmetry index [Wywiał (1981)]:



Figure 3. Age structure of Polish population in 2002

$$A = \frac{\gamma_1}{\sqrt{\gamma_2 + 2}} \quad (10)$$

where:

$$\gamma_1 = \frac{\mu_3}{\sigma^3} \quad (11)$$

$$\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3 \quad (12)$$

μ_2 , μ_3 , μ_4 — central moments of the 2nd, 3rd and 4th order, respectively, σ — standard deviation.

This index has the following property:

$$A \in \langle -1, 1 \rangle \quad (13)$$

The negative values of A indicate the left-sided asymmetry, the positive ones — right-sided asymmetry, and the zero value signifies the symmetry of distribution.

According to Cieślak(2003a) or Cieślak (2003b), the old-age index is defined as a function of arithmetic mean and asymmetry index:

$$S_i = M_i f(A_i) \quad (14)$$

where: S_i — old-age index in the i -th population,

M_i — arithmetic mean in the i -th population,

A_i — asymmetry index in the i -th population.

The final formula for the old-age index is:

$$S_i = M_i (-A_i + 2) \quad (15)$$

The old-age index is a value of arithmetic mean when the population has the «youngest» age structure, i.e. when asymmetry index is equal one. Further, the old-age index is the multiplicity of mean and measure of asymmetry.

Ageing or possible rejuvenation of age structure depends on the two factors: fertility and life span. The influence of combination of the two mentioned factors to ageing or rejuvenation process from the top or the bottom age pyramid is presented at table 1.

Table 1

AGEING PROCESS VERSUS CHANGES IN FERTILITY AND LIFE SPAN

Situation	Fertility	Process of ageing/rejuvenating		
		Life span		
		constant	fall	rise
		A	B	C
1	constant	unchanged	rejuvenation with loss	ageing at the top-with offset
2	fall	ageing at the bottom without offset	a) ageing at the bottom with loss or b) rejuvenation with loss	ageing at the bottom and at the top with offset
3	rise	rejuvenation without offset	rejuvenation with loss	a) unchanged orb) ageing at the top with offset orc) rejuvenation with offset

Source: Cieślak (2003a)

Situation presented in table 1 gives us various possibilities of interpretation of ageing process of population from the top or the bottom of population pyramid. Extended interpretation of considered process is given in Cieślak (2003a) and Cieślak (2003b).

4. Mortality functions, survival models and life tables

Ageing process is closely connected with mortality. When intensity of deaths at old age x is low, means that the person survive age x and becomes older. It means that ageing process can be measured also by searching mortality process. The classical mortality function is Gompertz deaths intensity function, which describe the probability of death at age x years as a function internal factors determining a resistance of human organism.

Gompertz mortality function³⁶ can be described as follows:

$$\mu(x) = R_0 e^{\alpha x} \quad (16)$$

Values of parameters function (16) can be also interpreted as measures of ageing process.

An extension of the classical Gompertz model is Gompertz-Makeham model:

$$\mu(x) = A + R_0 e^{\alpha x} \quad (17)$$

An interesting interpretation has parameter A , which describe the influence of external factor to the natural mortality process. Differences in parameter A in various populations can be interpreted as differences in ageing process among them.

Instead of mortality we can observe survival process. Survival model³⁷ is defined by the following functions:

— survival function which describe the probability of survive at least of t -years:

$$S(t) = P(T > t) \quad (18)$$

— dystrybuant:

$$F(t) = P(T \leq t) \quad (19)$$

— density function:

$$f(t) = \frac{dF(t)}{dt} \quad (20)$$

— hazard function:

$$\lambda(t) = \frac{f(t)}{S(t)} \quad (21)$$

An important problem is to determine the distribution of random variable T . In this case we can do the following:

³⁶ See for example Cieślak (1992).

³⁷ Survival model is vividly discussed in demography and actuarial mathematics. Among many publications see for example: Frątczak, Józwiak, Paszek (1996) or Ostasiewicz (2000).

— to search or to assume the analytical form of the distribution of life span T , we say in this case about parametric survival models,

— do not assume the analytical form of the distribution of life span, we have nonparametric survival models in this case.

Parametric survival models [Ostasiewicz (2000)] are based on the following distributions: uniform, exponential, Weibull, gamma, beta, normal, Pareto, Gompertz. There arise the problem which distribution to take into consideration in empirical investigation. This problem will not be discussed here. The second, nonparametric approach leads to construction of life tables. The problem is widely discussed in demography and actuarial mathematics [Hinde (1998)].

In conclusion it ought to be stressed, that ageing process cannot be investigated and interpreted only in terms of changing in age structure of population measured by various indices, but it is obvious to search it in wide context of fertility process, life span, mortality, hazard function and life tables.

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