

MODELING THE VALUES OF REFLEXIVE CHARACTERISTICS OF AGENTS WITHIN THE MANAGEMENT OF HERD BEHAVIOR AT THE ENTERPRISES

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The central place at the formation of agent behavior at enterprises is the study of decision-making procedures and factors that mediate their choice. To determine the values of the reflexive characteristics of agents in the framework of management of herd behavior at enterprises a new approach is proposed based on questionnaire methods, the apparatus of the theory of fuzzy sets and neural network modeling.

The determination of the values of reflexive characteristics of agents is carried out by the formation of fuzzy sets in the framework of the theory of L. Zadeh based on the results of a questionnaire of agents in selected areas. The agents are distributed by the Kohonen map into groups in order to numerically determine the values of their reflexive characteristics based on formed fuzzy sets. An important applied value in interpreting the results of the Kohonen SOM clustering is the ability to obtain representatives of specific clusters and average values of their characteristics, which are determined by the parameters of the network neurons and represent cluster centers. As a result of the clustering of input data vectors by directions of determining the values of the reflexive characteristics of agents, typical values of the required parameters are obtained for agents-representatives of clusters. The values of the reflexive characteristics of agents can be used to evaluate the results of decision-making by agents using the functions of reflexive choice to ensure effective management of manifestations of herd behavior at enterprises.

The proposed modeling methodology will allow to identify the prerequisites for the manifestation of herd behavior at enterprises and the potential circle of agents for the formation of adequate managing actions in the process of ensuring effective management of herd behavior and achieving the goals of the enterprise.

Keywords: *modeling, reflexive characteristics, agent, herd behavior, enterprise, questionnaires, fuzzy sets, self-organizing map*

JEL Classification: C02, C45, C52, C53, D91

Introduction

The study of decision-making procedures and the factors determining their choice occupies a central place in the formation of the behavior of agents at enterprises. In modern conditions of managing enterprises, which are determined by the instability of the external and internal environment and the need to take into account and process a large amount of information from various information sources. It becomes especially relevant to take into account irrational components in the decision-making process. If there is not enough information to make decisions or if there is no possibility of its independent processing, the decisions made by agents at enterprises can be based on the observed decisions or representations of other agents, which creates conditions for manifestations of herd behavior. The prerequisites for the manifestation of herding at enterprises are due to cognitive distortions in judgments, among which one can single out conformism, the effect of familiarity with the object, preference for zero risk, submission to authority, etc.

Indeed, the above cognitive distortions reflect the essence of reflexive processes in decision-making by economic agents in the process of herd behavior and affect not only the decision-making process, but also its result. Nobel laureates of different years have been investigating this issue. Thus, M. Allais was one of the first in his works to refute the rationality of the behavior of economic agents [1]. D. Kahneman and A. Tversky [2] proved that the same decision can have different values for agents with respect to any reference point (for example, from the initial state of the individual or from the situation in which the decision was made). R. Thaler studied cognitive distortions in judgments that manifest themselves in the decision-making process of economic agents and can lead to inaccurate judgments, incorrect interpretation of information or demonstration of irrationality in behavior and economic decision-making [3].

Such effects of irrational behavior and associated manifestations of herd behavior are actively studied by scientists in a variety of subject areas: in financial [4] and consumer's [5, 6] markets, bank lending [7],

cooperative behavior of people in society [8], etc. For example, in article [8] modeling of human behavior types with usage of Kohonen self-organizing maps is carried out on the basis of laboratory experiments “Public good” outcomes, and interpretation of the results in terms of “reciprocity” and “free-rider” behavioral hypotheses. The works of V. Danich and K. Shekhovtsova [9], S. Solodukhin [10], I. Stashkevych [11, 12] and others are devoted to studies of the influence of reflexive components on the decision-making process and related manifestations of herd behavior at the level of enterprises.

So, in [9] it is pointed out that there is an indirect impact on the activities of enterprises through fluctuations in exchange rates in financial markets and the associated currency panics and hype. S. Solodukhin conducted research on the manifestations of the herd behavior of agents in the internal and external environment of enterprises [10], in which the author substantiates the expediency of forming a base of typical models for studying the subject area. However, it should be noted that the models presented in [9, 10] are quite general, which implicitly take into account the reflexive characteristics of the decision-making process by agents. In addition, the authors do not divide the features of accounting for the selected characteristics within the proposed models and present them as generalized categories and parameters, which makes it difficult to use the models given in [9, 10] to solve the problems of managing herd behavior at enterprises.

In [11, 12] the authors proposed conceptual provisions for minimizing the resistance of personnel to organizational changes at enterprises, within the framework of which they developed: an approach to assessing the level of support for organizational changes in the team after information interaction between employees; approach to decision-making in the field of management by minimizing the resistance of personnel to organizational changes at the enterprise using elements of herd behavior based on a reflexive approach.

In [11] the authors considered the influence of conformism, as a special case of the manifestation of herd behavior, on the results of decision-making by agents, but did not present a methodology for

determining the values of reflexive characteristics of agents, which should be decisive in managing the manifestations of herd behavior in enterprises. Similarly, the works [13, 14] examines how the reflexive characteristics of agents affect the level of agreement of goals in the group without indicating specific methods for determining such characteristics.

Thus, due to the lack of effective mechanisms for identifying and numerically assessing the values of reflexive characteristics of agents, in order to determine the main parameters of the decision-making process by agents and effectively manage the manifestations of herd behavior at enterprises, it is necessary to develop an appropriate methodology using methods of economic and mathematical modeling.

The objective of the article is to develop a new method for assessing the values of reflexive characteristics of agents within the framework of management of herd behavior at enterprises using methods of economic and mathematical modeling.

Methodology

Understanding herd behavior as a result of decision-making strategy by management agents, which is not based on rational judgments, but is focused on imitating more authoritative and/or other economic agents when making decisions, the main reflexive components of the decision-making process by economic subjects are defined in [12-14]: awareness of agents, their competence, authority, propensity to imitate and intentional orientation.

Considering the qualitative nature of these categories, the subjectivity of their assessments and weak formalization, there is a need to use mathematical tools that are designed to process fuzzy, linguistically defined characteristics. An effective tool for solving such problems is the theory of fuzzy sets by L. Zadeh [15].

Moreover, each of these categories depends on many factors influencing it, the list of which and the degree of influence are also not determined by any objective circumstances and can be set by analysts depending on their own understanding of their essence. Dependence on the subjective opinion of individual experts can be

reduced by using special modeling methods that can identify patterns in the structure of an array of heterogeneous data when there are no predetermined values of the resulting indicator.

In such conditions, the cluster approach is the most suitable tool for searching for hidden patterns in sets of explanatory variables. As a result, it becomes possible to form fairly homogeneous groups of studied objects characterized by similar properties.

Consequently, within the framework of the toolkit for diagnosing manifestations of herd behavior, based on the fuzzy sets formed at the first stage, clustering is carried out both in order to determine the numerical values of the degree of awareness, competence, authority, propensity to imitate and intentional orientation of agents, and for the identification of a potential circle of agents on which the control actions of the mechanism of reflexive management of herd behavior will be directed.

The sequence of assessing the values of the characteristics of agents within the framework of our concept of modeling processes of reflexive management of herd behavior at enterprises includes the following main stages:

1. Questionnaire of agents according to directions of assessing the reflexive characteristics.
2. Formation of fuzzy sets for all characteristics based on the results of questioning agents.
3. Clustering agents according to directions of identifying reflexive characteristics.
4. Determination of the values of reflexive characteristics of agents and the potential circle of agents for reflexive management of herd behavior to achieve goals in the enterprise.

Let's consider each of the stages in more detail.

Due to the fact that in the process of manifestation of herd behavior the same agent can be both a managed and a managing, we will further use the term "management agent".

The determination of the values of the characteristics of the agents will be carried out by the method of questionnaires in the directions of identifying the relevant parameters within the framework of the object model of reflexive management of herd behavior at enterprises [13, 14].

To do this, questionnaires are initially drawn up according to five directions of identification:

α_{it} – agent's awareness of the i -th agent at the moment of time t ;

γ_{it} – competence of the i -th management agent at the moment of time t ;

β_{it} – authority of the i -th management agent at the moment t ;

ω_i – propensity to imitate the i -th agent;

v_{it} – estimates of intentions (the value of a particular decision for a particular management agent) of the i -th agent at the moment of time t .

Let the questionnaire for each of the directions includes K questions $k = \overline{1, K}$, each of which contains a list of L possible answers $l = \overline{1, L}$. At the same time, the questions of the questionnaire should be formed in such a way that the options for answers to them by the respondents (agents) decrease the value of the degree of a particular characteristic as they descend. Answers to questions should be formulated variably and unambiguously so, that the respondent can choose one of the listed options.

To compose questions in the area of revealing reflexive characteristics, one can use, for instance, the studies of the famous British psychologist R. Cattell [16] and D. McClelland competence tests [17]. In addition, we assign a linguistic term to each of the answer options, which characterizes the degree of approximation of the value of a particular characteristic to its maximum possible value. To formalize the degree of such membership and further use within the framework of the mechanism of reflexive management of herd behavior at enterprises [14], we apply the theory of fuzzy sets by L. Zadeh [15], which is widely used by researchers to solve various types of problems of intellectual data processing using linguistic variables and qualitative characteristics of research objects. L. Zadeh expanded the classical concept of a set, assuming that the characteristic function (membership function for a fuzzy set) can take any values in the interval $[0; 1]$.

To survey agents, questionnaires will be used that implement, respectively [14]:

methodology for determining the propensity to imitate and authority of agents, based on the modified structure of the questionnaire by R. Cattell [16];

methodology for determining the competence of management agents in enterprises based on the modified structure of the D. McClelland questionnaire [17], as well as competence clusters of L. Spencer and S. Spencer [18];

a method for determining the value of a decision for a management agent based on the prospect theory of D. Kahneman and A. Tversky [2] and considering a decision with respect to any reference point (the personal well-being of the agent and the enterprise);

a method of assessing awareness, which is based on identifying the degree of reliable knowledge of the agent about the actual state of the object (alternatives of the decision) and the circumstances affecting it.

To determine the propensity to imitate and the authority of agents, we will use the elements of the 16-factor personality model of R. Cattell [16]. The British psychologist R. Cattell in his research proposed a model of personality and created an appropriate psychodiagnostic technique based on a questionnaire that allows you to evaluate each of these factors. In the framework of our study, we will use elements of the methodology of R. Cattell (form A), which allows us to determine the specificity of the character, inclinations and interests of the individual. At the same time, to assess the agent's propensity to imitate, we will use the adapted questionnaires of R. Cattell according to the clusters identified within the framework of the methodology, which correspond to the characteristics of the agents' behavior:

cluster E, by means of which the conformity / dominance of the agent is assessed (adapted for use in determining the values of reflexive characteristics of agents in the process of managing herd behavior in enterprises; adaptation consists in changing the direction of assessing the results of the survey, i.e. the highest value of points is interpreted as a high degree of propensity to imitate, and the least – as low);

cluster Q1, by means of which the conservatism / radicalism of the agent is assessed (note that the increased degree of radicalism of the individual contributes to the rapid decision-making on imitation);

cluster Q2, by means of which the dependence on the group / self-sufficiency is assessed (similarly to cluster E, it is adapted for use in determining the values of reflexive characteristics of agents in managing herd behavior at enterprises; adaptation consists in changing the direction of assessing the results of the survey, i.e. the highest score interpreted as a high degree of dependence on the group, and the smallest – as low).

So, to assess the agent’s propensity to imitate in the decision-making process, the questionnaires contain $k_{\omega} = \overline{1,33}$ questions, each of which includes a list of possible answers $l_{k_{\omega}} = \overline{1,3}$, according to the method adapted by R. Cattell. In this case, the function $\mu(x^{k_{\omega}l_{k_{\omega}}})$ takes values in some linearly ordered set of memberships $M_{\Omega} = \{0; 0.5; 1\}$. For example, for question 28 of the questionnaire about the propensity to imitate “It is easier for me to solve a difficult question or problem”:

$$\mu(x^{28_{\omega}l_{28_{\omega}}}) = \begin{cases} 0, & l_{28_{\omega}} = 1 \mid \text{if I think about them alone;} \\ 0.5, & l_{28_{\omega}} = 2 \mid \text{right in the middle;} \\ 1, & l_{28_{\omega}} = 3 \mid \text{if I discuss them with others.} \end{cases}$$

Thus, the set Ω contains a complete list of membership functions for all 33 questions and answer options to guide the identification of the propensity to imitate management agents.

Similarly, fuzzy sets are formed to assess the authority of agents within the framework of the mechanism of reflexive management of herd behavior at enterprises. At the same time, to assess the authority of the agent, adapted questionnaires by R. Cattell according to the 16-factor model of personality (Form A of the questionnaire) are used, namely questions corresponding to:

cluster Q3, with the help of which low / high conceit of the agent is estimated;

cluster E, which evaluates the conformity / dominance of the agent (the highest score is interpreted as a high degree of dominance in the group, and the lowest – as low).

According to the adapted method of R. Cattell, the developed questionnaires contain $k_\beta = \overline{1,3}$ questions regarding the assessment of the authority of agents, each of which contains a list of possible answers $l_{k_\beta} = \overline{1,3}$. The function $\mu(x^{k_\beta k_\beta})$ takes values in some linearly ordered set $M_B = \{0; 0.5; 1\}$. The set B contains a complete list of membership functions for all questions and answer options to guide the identification of the authority of management agents.

We will determine the competence of management agents at enterprises using the modified structure of the D. McClelland questionnaire [17] and competence clusters of L. Spencer and S. Spencer [18]. Here, to assess the competence of agents, the possibility of using generalized competence models for technical specialists, proposed in [18] as a development of the ideas of D. McClelland, is considered. The model highlights key competencies, among which the most significant are: achievement, impact and influence, conceptual and analytical thinking and initiative. Thus, to assess the competence of agents, the questionnaires contain $k_\gamma = \overline{1,19}$ questions, each of which includes a list of possible answers $l_{k_\gamma} = \overline{1,5}$. Accordingly, the function

$\mu(x^{k_\gamma k_\gamma})$ takes values in some linearly ordered set $M_\gamma \in [0;1]$ from the membership set $M_\gamma = \{0; 0.25; 0.5; 0.75; 1\}$. The set Y contains a complete list of membership functions for all questions and answer options to identify the competence of agents.

Questionnaires and corresponding fuzzy sets in the direction of awareness contain questions and corresponding answer options developed on the basis of understanding the definition of agent awareness, which implies the degree of his reliable knowledge about the actual state of the object (alternatives of the decision being made) and the circumstances affecting it, and contain $k_\alpha = \overline{1,8}$ questions, each of which includes a list of possible answers $l_{k_\alpha} = \overline{1,5}$.

At the same time, the key factors are: the sources of obtaining information, the reliability of the facts and information used in the

decision-making process, the frequency of using distorted information, the completeness and efficiency of obtaining the necessary information in the decision-making process and the degree of general awareness of the agent about the decision being made. The degrees of belonging are described as follows: for example, for the first question of the agent’s awareness assessment questionnaire:

$$\mu(x^{l_{1\alpha}}) = \begin{cases} 0, & l_{1\alpha} = 1 & \left| \begin{array}{l} \textit{I don't use sources. I use only existing} \\ \textit{knowledge;} \end{array} \right. \\ 0.25, & l_{1\alpha} = 2 & \left| \begin{array}{l} \textit{information from management;} \\ \textit{information from management and} \\ \textit{which is discussed with employees;} \end{array} \right. \\ 0.5, & l_{1\alpha} = 3 & \left| \begin{array}{l} \textit{information from management and} \\ \textit{official information resources on the topic;} \end{array} \right. \\ 0.75, & l_{1\alpha} = 4 & \left| \begin{array}{l} \textit{studied the topic in detail using all} \\ \textit{available official information resources.} \end{array} \right. \\ 1, & l_{1\alpha} = 5 & \end{cases}$$

Moreover, in terms of the mechanism of reflexive management of herd behavior at enterprises, the membership function $\mu(x^{k_{a'l_{k\alpha}}})$ expresses how agent i is informed at the moment of time t . The function $\mu(x^{k_{a'l_{k\alpha}}})$ takes values in some linearly ordered set $M_A \in [0;1]$ and consists of five elements according to the number of possible answers in each of the questions: $M_A = \{0; 0.25; 0.5; 0.75; 1\}$. The set A contains a complete list of membership functions for all questions and answer options to identify the awareness of management agents.

In the same way, fuzzy sets are formed for the questions of questionnaires to determine the intentional orientation of agents (the value of the decision made for management agents). In this case, the value of particular decision for the agent is estimated according to the

prospect theory of D. Kahneman and A. Tversky [2] and is considered relative to any reference point (the personal well-being of the agent or the enterprise).

Due to the fact that the theory of fuzzy sets in a certain sense is reduced to the theory of probability [15], the value of the membership function can be considered as the probability of covering an element, for example $x^{k_\nu l_{k_\nu}}$, by some random set. Then, from the point of view of the mechanism of reflexive management of herd behavior at enterprises $\mu(x^{k_\alpha l_{k_\alpha}})$ can be interpreted as the probability that agent will make a choice of some decision.

At the same time, under the fuzzy set A of the degree of awareness of agents at a moment t we will understand the set of ordered pairs made up of elements $x^{k_\alpha l_{k_\alpha}}$ of the universal set X_α and the corresponding degrees of membership $\mu(x^{k_\alpha l_{k_\alpha}})$:

$$A = \left\{ \left(x^{k_\alpha l_{k_\alpha}}, \mu(x^{k_\alpha l_{k_\alpha}}) \right) \mid x^{k_\alpha l_{k_\alpha}} \in X_\alpha, \mu(x^{k_\alpha l_{k_\alpha}}) \in M_A \right\}. \tag{1}$$

If $M_A = \{0,1\}$, that is, it consists of only two elements, then the fuzzy set can be considered as an ordinary crisp set.

Similarly, fuzzy sets will be formed for each of the other selected reflexive characteristics of agents within the framework of the mechanism of reflexive management of herd behavior at enterprises.

So, to determine the competence of the management agent, we get the set:

$$Y = \left\{ \left(x^{k_\gamma l_{k_\gamma}}, \mu(x^{k_\gamma l_{k_\gamma}}) \right) \mid x^{k_\gamma l_{k_\gamma}} \in X_\gamma, \mu(x^{k_\gamma l_{k_\gamma}}) \in M_Y \right\}, \tag{2}$$

where X_γ is the universal set of $x^{k_\gamma l_{k_\gamma}}$.

To determine the authority of agents, we get a set:

$$B = \left\{ \left(x^{k_\beta l_{k_\beta}}, \mu(x^{k_\beta l_{k_\beta}}) \right) \mid x^{k_\beta l_{k_\beta}} \in X_\beta, \mu(x^{k_\beta l_{k_\beta}}) \in M_B \right\}, \tag{3}$$

where X_β is the universal set of $x^{k_\beta l_{k_\beta}}$.

To determine the propensity of agents to imitate in the decision-making process, we obtain the following set:

$$\Omega = \left\{ \left(x^{k_\omega l_{k_\omega}}, \mu(x^{k_\omega l_{k_\omega}}) \right) \mid x^{k_\omega l_{k_\omega}} \in X_\omega, \mu(x^{k_\omega l_{k_\omega}}) \in M_\Omega \right\}, \quad (4)$$

where X_ω is the universal set of $x^{k_\omega l_{k_\omega}}$.

In turn, to determine the intentional orientation of agents in the decision-making process at the moment of time t , we obtain a set:

$$I = \left\{ \left(x^{k_\nu l_{k_\nu}}, \mu(x^{k_\nu l_{k_\nu}}) \right) \mid x^{k_\nu l_{k_\nu}} \in X_\nu, \mu(x^{k_\nu l_{k_\nu}}) \in M_I \right\}, \quad (5)$$

where X_ν is the universal set of $x^{k_\nu l_{k_\nu}}$.

So, for example, according to the results of the survey of the i -th agent A_i , his awareness α_{it} at the moment of time t will be determined by the answers $x^{k_\alpha l_{k_\alpha}^*}$, $k_\alpha = \overline{1, 8}$, where the parameter $l_{k_\alpha}^*$ will be the answer chosen by the agent from the list of options L_α to the question k_α :

$$A^* = \left\{ x_{it}^{k_\alpha l_{k_\alpha}^*} \in X_\alpha \right\}, k = \overline{1, K_\alpha}. \quad (6)$$

Then, the set of fuzzy representations of the results of the questionnaire in the direction of identifying the degree of awareness of the agent will have the following form:

$$\tilde{A} = \left\{ \mu(x_{it}^{k_\alpha l_{k_\alpha}^*}) \mid x_{it}^{k_\alpha l_{k_\alpha}^*} \in X_\alpha \right\}. \quad (7)$$

In this case, the set of fuzzy sets of agent survey results in all directions of revealing the reflexive characteristics of agents will correspond to the sets $\tilde{A}, \tilde{Y}, \tilde{B}, \tilde{Q}, \tilde{I}$ (Table 1).

Table 1

SETS OF FUZZY REPRESENTATIONS OF THE RESULTS OF QUESTIONNAIRES OF REFLEXIVE CHARACTERISTICS OF AGENTS

Reflexive characteristics	Sets of survey results	Sets of fuzzy representations of survey results
α_{it} – i -th agent’s awareness at the moment t	$A^* = \left\{ x_{it}^{k_\alpha l_{k_\alpha}^*} \in X_\alpha \right\}$	$\tilde{A} = \left\{ \mu(x_{it}^{k_\alpha l_{k_\alpha}^*}) \mid x_{it}^{k_\alpha l_{k_\alpha}^*} \in X_\alpha \right\}$
γ_{it} – competence of a management agent i at the moment t	$\gamma^{*} = \left\{ x_{it}^{k_\gamma l_{k_\gamma}^*} \in X_\gamma \right\}$	$\tilde{\gamma} = \left\{ \mu(x_{it}^{k_\gamma l_{k_\gamma}^*}) \mid x_{it}^{k_\gamma l_{k_\gamma}^*} \in X_\gamma \right\}$
β_{it} – authority of the management agent i at the moment t	$B^* = \left\{ x_{it}^{k_\beta l_{k_\beta}^*} \in X_\beta \right\}$	$\tilde{B} = \left\{ \mu(x_{it}^{k_\beta l_{k_\beta}^*}) \mid x_{it}^{k_\beta l_{k_\beta}^*} \in X_\beta \right\}$
ω_i – propensity to imitate of a management agent i	$\Omega^* = \left\{ x_i^{k_\omega l_{k_\omega}^*} \in X_\omega \right\}$	$\tilde{\Omega} = \left\{ \mu(x_i^{k_\omega l_{k_\omega}^*}) \mid x_i^{k_\omega l_{k_\omega}^*} \in X_\omega \right\}$
ν_{it} – assessment of intentions of i -th agent at the moment t	$I^* = \left\{ x_{it}^{k_\nu l_{k_\nu}^*} \in X_\nu \right\}$	$\tilde{I} = \left\{ \mu(x_{it}^{k_\nu l_{k_\nu}^*}) \mid x_{it}^{k_\nu l_{k_\nu}^*} \in X_\nu \right\}$

After obtaining the results of identifying the reflexive characteristics of agents to determine the degree of awareness, competence, value of a decision for a particular agent at the time t , the authority of agents and the propensity to imitate, the formed fuzzy sets must be grouped and processed so that the numerical values of the indicated parameters for each agent can be determined. And since for the task of assessing the reflexive characteristics of agents there is no generally accepted indicator and scale of its measurements, there is a need to solve the problem of searching for patterns of behavior of different groups of agents and regularities in their reactions to different influences. This problem is solved by clustering the obtained fuzzy values of reflexive characteristics in the relevant areas of analysis of the decision-making process. It should be noted that the purpose of such a clustering will be not only the numerical determination of the reflexive characteristics of agents, but also the

identification of a potential circle of agents on which the control actions of the mechanism of reflexive management of herd behavior will be directed.

There is a wide range of cluster analysis methods: K-means [19], K-medoids [20], Principal Component Analysis (PCA) [21], t-Stochastic Neighbour Embedding (t-SNE) [22], Dendrogram Method [23], Dendrite Method [24], Density-Based Spatial Clustering of Applications with Noise (DBSCAN) [25], Uniform Manifold Approximation and Projection (UMAP) [26], Balanced Iterative Reducing and Clustering Using Hierarchies (BIRCH) [27], Self-Organizing Maps (SOM) [28-30] and its variation – SOM algorithm with C-Weighted Medoids for dissimilarity data (RBSOM-CWMdd) and Batch SOM algorithm with Adaptive Heuristic C-Weighted Medoids for dissimilarity data (RBSOM-ACWMdd) [31], etc.

Each of these methods has its own advantages and areas of application and tasks where it works best. Experimental comparisons of the effectiveness of various clustering methods are described, in particular, in the papers [31-34].

Considering the capabilities of each of the mentioned methods, the small size of the database and the fuzzy nature of the analyzed indicators, in this study the toolkit of Kohonen self-organizing maps was chosen, which, in addition to the formation of homogeneous groups of studied objects, provides a convenient tool for visual analysis of clustering results. In particular, unlike other clustering methods, the location of an object on the Kohonen map indicates to the analyst how developed the characteristic under study is in comparison with others, since the best and worst objects according to the analyzed reflexive characteristic are located in opposite corners of the SOM.

Thus, to assessing the values of reflexive characteristics in terms of determining the degree of awareness of agents, competence, authority, propensity to imitate and intentional orientation, we will use the Kohonen neural network. In this case, we will supply the values of membership functions to fuzzy sets of the results of questioning agents to the inputs of the neural network.

A self-organizing map is a neural network without feedback, which is configured using an unsupervised learning algorithm by

identifying unknown patterns and structures in the given indicators of the objects under study. The learning algorithm provides a mapping of a high-dimensional space onto a low-dimensional map, while maintaining its topological structure. The topology-preserving property means that, as a result of self-organization, similar input data vectors are projected onto neurons located close to each other on the Kohonen map.

A self-organizing map is formed from neurons, each of which is connected to all neurons of the input layer (Fig. 1) [28-30].

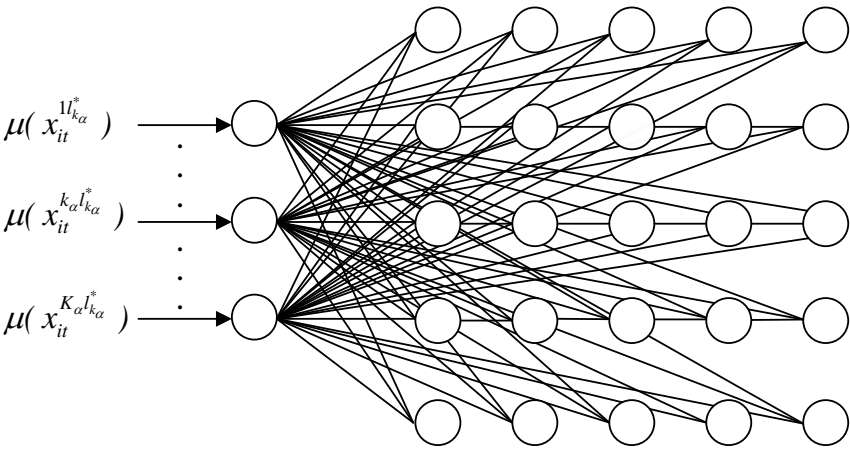


Fig. 1. The structure of the Kohonen neural network

In self-organizing maps, neurons are located at the nodes of a lattice, most often one- or two-dimensional. When constructing a two-dimensional lattice, an orthogonal or hexagonal structure is usually chosen (the hexagonal ordering of neurons on the Kohonen map makes it possible to visually represent the result of clustering objects more qualitatively).

The neurons of the input layer do not convert the input signals – they only transmit them to all elements of the self-organizing map. Each neuron of the Kohonen layer receives information regarding the

object of study in the form of a vector consisting of sets obtained as a result of questioning the studied agents $\tilde{A} = \left\{ \mu(x_{it}^{k_a^* k_\alpha}) \mid x_{it}^{k_a^* k_\alpha} \in X_\alpha \right\}$.

When a new data vector arrives at the input layer of the network, all neurons of the self-organizing map compete to become the winner. As a result of such a competition, the winner is the neuron that is most similar to the input data vector. According to [28], the measure of such similarity of the data vector to each neuron can be determined by calculating the Euclidean distance:

$$\|x - w_\alpha^j\| = \sqrt{\sum_{k_\alpha=1}^{K_\alpha} (x_{it}^{k_a^* k_\alpha} - w_{k_\alpha}^j)^2}, \quad j = \overline{1, M}, \quad (8)$$

where x is an input vector that consists of answers to the questions of the agents' questionnaire $\left\{ \mu(x_{it}^{k_a^* k_\alpha}); \dots; \mu(x_{it}^{k_a^* k_\alpha}); \dots; \mu(x_{it}^{K_a^* k_\alpha}) \right\}$;

w_α^j is a parameter vector of the i -th neuron of the Kohonen map, which consists of elements $\left\{ w_1^j, \dots, w_{k_\alpha}^j, \dots, w_{K_\alpha}^j \right\}$;

M is the number of neurons in the Kohonen map.

The winner in such a competition of self-organizing map neurons is one neuron that is most similar to the vector of input data according to the Euclidean distance (8). Its output will be one, the states of all other neurons of the SOM are equal to zero:

$$y_j = \begin{cases} 1, & \|x - w_\alpha^j\| = \min_{m=1, M} \|x - w_\alpha^m\| \\ 0, & \|x - w_\alpha^j\| \neq \min_{m=1, M} \|x - w_\alpha^m\| \end{cases}, \quad j = \overline{1, M} \quad (9)$$

Function (9) implements the “winner-take-all” rule of competition [28, 35]. After finding the neuron-winner with respect to the input data vector, its parameters and the neurons closest to it are adjusted in a certain neighborhood in the direction of the input vector, taking into account the coefficient of learning rate $\eta(\tau)$ and the function of the distance to the winner $h_{oj}(\tau)$ [28–30, 35–37]:

$$w_{\alpha}^j(\tau+1) = w_{\alpha}^j(\tau) + \eta(\tau) \cdot h_{oj}(\tau) \cdot [x(\tau) - w_{\alpha}^j(\tau)], \quad j = \overline{1, M}, \quad (10)$$

$$h_{oj}(\tau) = \exp \left[-\frac{\|r_o - r_j\|^2}{2 \cdot \sigma^2(\tau)} \right], \quad (11)$$

where r_o, r_j are the coordinates of the neuron-winner o and the j -th neuron (on the map);

$\sigma(\tau)$ is an effective width of the topological region (a specially chosen monotonically decreasing function of iteration number τ , for example, a linear or exponential).

As training progresses, the size of the topological region gradually decreases, and each new input data vector affects an ever smaller number of neurons. At the end of training, the parameters of only the nearest neighbors of the neuron-winner can be modified, and possibly only the winner itself.

The result of the tuning process will be the calculation of the parameters of the Kohonen layer neurons, which will correspond to various examples from the training set. In this way, the structure of the Kohonen map self-organizes, which acquires the ability to combine multidimensional data vectors into clusters by detecting similar statistical characteristics in them. As a result, the initial high-dimensional space is projected onto a two-dimensional map. Because self-organizing maps are characterized by the generalization property, they can recognize input examples that were not used when they were trained – the new input data vector corresponds to the map element on which it is mapped.

An important applied value of interpreting the results of Kohonen SOM clustering is the possibility of obtaining representatives of specific clusters and calculating their average values, which are determined by the network neurons parameters and represent the centers of clusters. Next, the value of the corresponding reflexive characteristics of each of the agents in the obtained clusters after processing by the neural network will be given the value of the parameters of the neuron that determines the center of the cluster. Thus,

as a result of the classification of the input vectors of data according to the directions of determining the values of the reflexive characteristics of the agents, we will get the typical values of the required parameters for agents representing a particular cluster [12-14]. These clusters correspond to different patterns of agents' characteristics, which provides reasoned grounds for considering the clusters to correspond to individual classes of agents' reflexive behavior.

Thus, as a result of neural network processing of data from agent questionnaires in the direction of determining the degree of awareness of agents, we will obtain a set of parameter values $\overline{A} = \{\alpha_{it}\}, i = \overline{1, N}$. It was previously determined that from the point of view of the concept of reflexive management of herd behavior at enterprises, we will interpret the parameters $x_{it}^{k\alpha l_{k\alpha}}$ as the probability, that at the moment of time t the agent i is fully informed about the area of the decision being made. In this case, the translation of the obtained values α_{it} into the initial metric of linguistic parameters is not necessary.

Similarly to the considered example of clustering using self-organizing maps, after processing the results of the questionnaire in the directions of determining competence, authority, propensity to imitate and intentional directions (decision value) of agents, we obtain the following sets:

$\overline{Y} = \{\gamma_{it}\}, i = \overline{1, N}$ – set of values that determine the competence of management agents at the moment of time t ;

$\overline{B} = \{\beta_{it}\}, i = \overline{1, N}$ – a set of values that determine the authority of management agents at the time t ;

$\overline{\Omega} = \{\omega_{it}\}, i = \overline{1, N}$ – a set of values that determine the propensity to imitate agents;

$\overline{I} = \{v_{it}\}, i = \overline{1, N}$ – from the point of view of the concept of reflexive management of herd behavior at enterprises, v_{it} can be interpreted as an intentional orientation to make a choice of a certain decision for a specific agent i (or as a probability that at the moment t agent i is ready to make a choice of certain decision).

Experiment results

We will test the proposed approach for calculating the reflexive characteristics of agents in the framework of managing herd behavior at Novokramatorsky Mashinostroitelny Zavod using the example of agreeing on a decision on the choice of a commercial offer for suppliers of components for the manufacture of an electric overhead foundry crane.

The manufacturing of engineering products, in particular cranes, is a science-intensive and expensive process. At the same time, on average, one engineering product of the plant accounts for about 50% of purchased components. To organize such purchases for each of the projects, tenders are held for suppliers and options for commercial proposals are drawn up for the full list of necessary components for the order. Thus, the decision by the tender commission to select suppliers for the purchase of components for the production of a crane from two options of commercial proposals is an extremely important decision due to the high cost of purchased parts, which necessitates the assessment of the reflexive characteristics of agents, including for the management of the agents' decision-making process.

The tender commission consists of: 1 chairman of the tender commission; 3 representatives of the audit commission; 1 accounting representative; 1 lead design engineer; 1 constructor of the 1st category; 2 representatives of the pricing and cost management department; representatives of the procurement department in the areas of purchase of components, including 3 representatives of the bureau of purchase of electrical equipment, 2 representatives of the bureau of purchase of gearboxes, 2 representatives of the bureau of purchase of hardware products, 2 representatives of the bureau of purchase of rubber products, 2 representatives of the bureau of purchase of bearings.

For the convenience of carrying out calculations and analyzing the results obtained, we denote the agents conditionally as Agent 1, Agent 2, etc. up to 20 by the number of agents participating in the approval of commercial proposals, and assign them the appropriate indexes $i = 1, 20$.

The data of the survey of agents regarding the decision to choose a commercial proposal for the purchase of components for an electric overhead foundry crane, according to the questionnaires in the direction of identifying the degree of awareness of agents, converted into membership functions, are given in Table 2.

Table 2

THE RESULTS OF THE SURVEY OF AGENTS IN THE DIRECTION OF IDENTIFYING THE REFLEXIVE CHARACTERISTIC "AWARENESS" (SET A^*)

$\begin{matrix} \text{№ of question} \\ k_{\alpha} \\ \text{Agent } i \end{matrix}$	1	2	3	4	5	6	7	8
Agent 1	0	0.25	0.75	0.5	0.75	0.5	0.75	1
Agent 2	1	1	1	0.5	0.5	0	0.75	0.5
Agent 3	0.5	0.75	1	0.5	0.75	1	1	1
Agent 4	0	0.5	1	0.75	0.75	0.75	0.75	1
Agent 5	1	1	0.5	0.75	0.75	0.25	0.75	0.25
Agent 6	0.75	1	1	0.75	0.5	0	0.5	0.5
Agent 7	0	0.75	1	0.75	0	0.75	0.25	0.5
Agent 8	1	0	1	1	0.25	0.25	0.75	1
Agent 9	1	1	0.75	0.5	0.5	0.5	0	0.75
Agent 10	1	1	0.5	0.5	1	0.5	0.75	1
Agent 11	0.5	0.75	0.75	0.75	0.75	0.5	0.5	1
Agent 12	0.5	0.75	1	1	0.5	0.75	1	0.25
Agent 13	1	0.25	1	0.5	0	0.75	0.5	0.5
Agent 14	1	0	0.75	0	1	1	0.5	0.75
Agent 15	0.5	0.75	1	0.75	0.75	0.5	0.75	0.5
Agent 16	0.25	0.5	0.75	0.75	0.75	0.5	0.5	0.75
Agent 17	0.75	0.25	0.75	0.5	0.5	0.75	0.5	1
Agent 18	0.75	0.25	1	0.75	1	0.25	0	0.5
Agent 19	0.5	0.75	0.5	0.75	1	0.25	0.75	0.5
Agent 20	0.5	0.25	0	1	1	0.75	0.5	1

Similar tables based on the results of questioning agents are calculated for all areas of revealing the reflexive characteristics of agents. Note, that since membership functions describe possible answers ranging from 0 to 1, then preliminary data normalization is not required for further calculations and analysis.

In the framework of this study, to determine the values of the reflexive characteristics of agents in the process of managing herd behavior at enterprises using Kohonen maps, we will use the analytical platform Deductor Studio. Fig. 2 shows the self-organizing map built on the basis of the data of Table 2, classifying agents according to their answers to questionnaires in the direction of revealing the reflexive characteristics “awareness”.

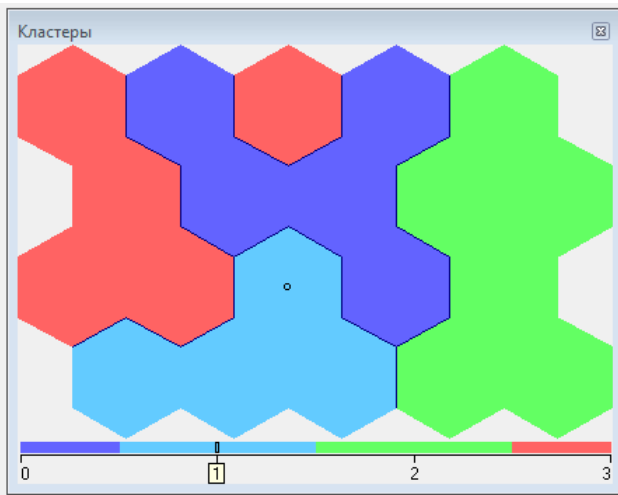


Fig. 2. Clustering of agent survey results in the direction of awareness

As a result of clustering using the Kohonen map, 4 clusters were obtained (0–3, the numbering of which can be seen from the coloring on the horizontal bottom line).

Let us analyze the distribution of control agents between clusters. As can be seen in Fig. 3, there are 4 agents in cluster 0: 9, 10, 14 and 17.

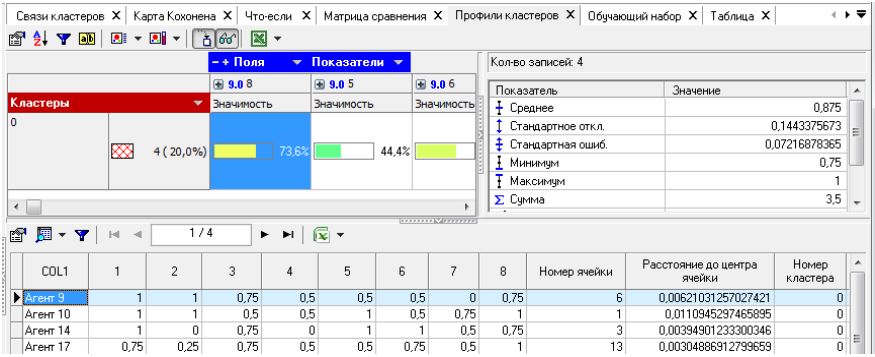


Fig. 3. Characteristics of the cluster 0 of SOM clustering results in the direction of revealing agent awareness

So, according to the proposed method, to numerically determine the awareness of agents, we assign the average value among all indicators, characterizing awareness, for agents who are included in the cluster. Then the value of the required reflexive characteristic “awareness” for each of the cluster agents will be $\alpha_{9t} = \alpha_{10t} = \alpha_{14t} = \alpha_{17t} = 0,875$.

Similarly, 4 agents hit into cluster 1: 7, 8, 12, and 13. Accordingly, we assign the average value of all membership functions over the cluster to the awareness values of these four agents $\alpha_{7t} = \alpha_{8t} = \alpha_{12t} = \alpha_{13t} = 0,5625$. Cluster 2 includes 6 agents: 1, 3, 4, 11, 16 and 20, whose awareness values will be assigned the cluster average $\alpha_{1t} = \alpha_{3t} = \alpha_{4t} = \alpha_{11t} = \alpha_{16t} = \alpha_{20t} = 0,9583$. Cluster 3 includes 6 agents: 2, 5, 6, 15, 18, and 19. Accordingly, we assign $\alpha_{2t} = \alpha_{5t} = \alpha_{6t} = \alpha_{15t} = \alpha_{18t} = \alpha_{19t} = 0,4583$ to the values of their awareness characteristics.

The results of clustering with self-organizing maps to determine the competence, the authority, the propensity of agents to imitate and decision value of agents according to the results of the survey are shown in Fig. 4.

A summary table of the values of reflexive characteristics of agents based on the results of a survey at Novokramatorsky Mashinostroitelny Zavod is presented in Table 3.

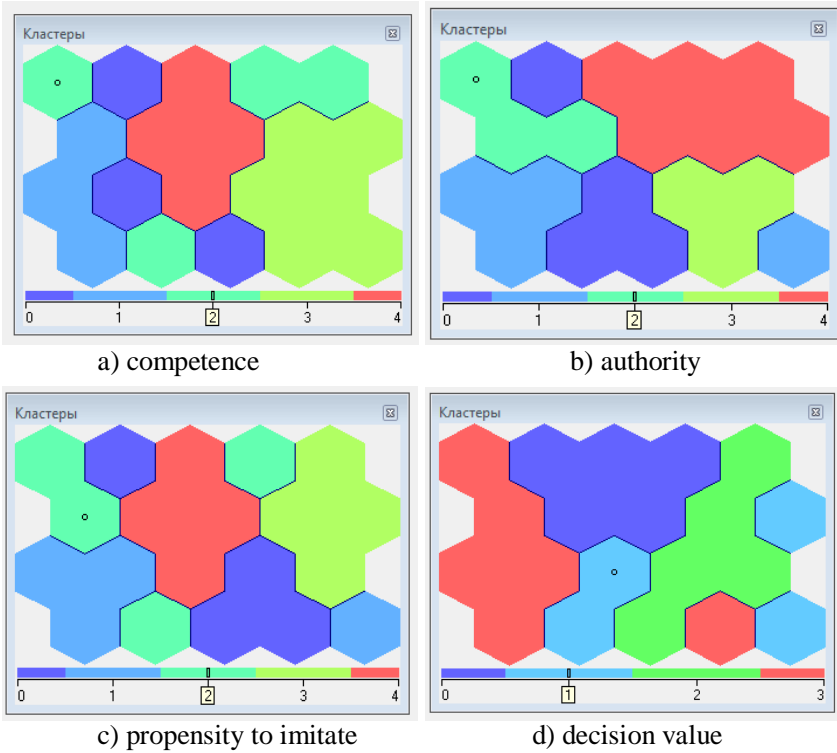


Fig. 4. The results of clustering by the self-organizing maps according to the indicators of reflexive characteristics of agents

In Table 3, agent 1 corresponds to the chairman of the tender commission. The values of his reflexive characteristics indicate that he enjoys authority (authority $\beta_{1t} = 0,83$), is competent (competence $\gamma_{1t} = 0,8$), is interested in choosing commercial offer No. 1 (decision value $v_{1t} = 0,7083$), is sufficiently informed (awareness $\alpha_{1t} = 0,9583$) and his propensity to imitate is quite low ($\omega_{1t} = 0,625$). Agent 1 in the considered example is the managing center, which is interested in the collegial acceptance of commercial offer No. 1 for the organization of the purchase of components for the crane by all agents.

Table 3

VALUES OF REFLEXIVE CHARACTERISTICS OF AGENTS OBTAINED AS A RESULT OF DATA PROCESSING BY KOHONEN MAPS

Agent i	Awareness $\bar{A} = \{\alpha_{ii}\}$	Competence $\bar{Y} = \{\gamma_{ii}\}$	Authority $\bar{B} = \{\beta_{ii}\}$	Propensity to imitate $\bar{\Omega} = \{\omega_i\}$	Decision Value $\bar{I} = \{v_{ii}\}$
Agent 1	0.9583	0.8	0.83	0.625	0.7083
Agent 2	0.4583	0.375	0.167	0.9	0.15
Agent 3	0.9583	0.375	0.167	0.4	0.65
Agent 4	0.9583	0.8	1	0.9	0.65
Agent 5	0.4583	0.67	0.625	1	0.5625
Agent 6	0.4583	0.8	0.5	0.625	0.7083
Agent 7	0.5625	0.167	1	0.4	0.5625
Agent 8	0.5625	0.8	1	0.625	0.7083
Agent 9	0.875	0.8	0.167	0.5	0.65
Agent 10	0.875	0.167	0.625	0.9	0.65
Agent 11	0.9583	0.375	0.625	0.5	0.15
Agent 12	0.5625	0.375	0.167	1	0.15
Agent 13	0.5625	0.375	1	0.9	0.5625
Agent 14	0.875	0.75	0.83	0.4	0.7083
Agent 15	0.4583	0.167	0.83	0.625	0.5625
Agent 16	0.9583	0.75	0.167	1	0.15
Agent 17	0.875	0.75	0.5	0.4	0.65
Agent 18	0.4583	0.67	0.167	0.5	0.15
Agent 19	0.4583	0.67	0.625	0.9	0.7083
Agent 20	0.9583	0.375	1	0.4	0.7083

As an agent-leader, to whom agent 1 should direct managerial influences to ensure that other agents follow him, based on the analysis of the obtained reflexive characteristics of agents (Table 3) in

accordance with the methodology [12-14], it is advisable to choose such an agent who has the value of the reflexive characteristics of authority and competence as close as possible to 1.

In Table 3 agent 1 represents the managing center, agent 4 acts as a leader agent ($\beta_{4t} = 1$, $\gamma_{4t} = 0,8$), to whom managerial influences must be directed in order to provide a signal to agents inclined to imitation that this agent can be trusted in the correctness of the decisions made and, accordingly, he can be imitated. Consequently, reflexive managerial influences will be directed at him to start the mechanism of imitation among other agents.

Thus, the obtained values of the reflexive characteristics of agents in the areas of detection can be used to select a circle of agents for the realization of reflexive managerial actions to achieve the goals of enterprise management, as well as to forecast the results of decision-making by agents about the choice of alternatives using the functions of reflexive choice, the possibility of applying which are described in detail in [14].

Conclusions

To determine the values of reflexive characteristics of agents in the article it is proposed to use a complex of models developed on the basis of survey methods, the apparatus of fuzzy set theory by L. Zadeh, and neural network modeling. The determination of the values of reflexive characteristics of agents (awareness, competence, authority, propensity to imitate and decision value) is carried out by forming fuzzy sets within the framework of the theory of L. Zadeh and calculating the properties of agents according to the results of questionnaires in selected areas.

Based on the membership of agents to corresponding fuzzy sets, the Kohonen map groups agents into clusters with the aim of determination of the values of their reflexive characteristics. In this case, as a result of clustering with Kohonen maps, a potential circle of agents is revealed, on which the managing effects of the mechanism of reflexive management of herd behavior will be directed.

An important applied value of interpreting the results of SOM clustering is the possibility of obtaining both representatives of specific classes and average values of the characteristics of class representatives, which are determined by the parameters of the network neurons and represent cluster centers. The reflexive characteristics of each of the agents in the resulting clusters after processing by the neural network are proposed to be assigned the values of the parameters of the neuron that determines the cluster center. Thus, as a result of clustering the input data vectors along the directions for determining the values of the reflexive characteristics of agents, typical values of the required parameters for agents representing classes are obtained.

These values can be used within the framework of the mechanism for diagnosing the manifestations of herd behavior in enterprises, for managing the decision-making process of agents and for predicting the behavior of agents using the functions of reflexive choice.

Promising direction of research is in the development of individual diagnostic mechanisms and a system of practical recommendations for the reflexive management of herd behavior regarding managerial decisions at meetings of various levels of enterprise management, decisions on choosing suppliers and consumers of enterprise products, counteracting staff resistance to organizational changes.

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