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LYAPUNOV EXPONENT FOR THE CONSTRUCTION OF CRISIS PHENOMENA PRECURSORS AT STOCK MARKETS

Complex systems, regardless of nature, exhibit nonlinear characteristics that include both deterministic and stochastic components. These systems generate signals that exhibit complex characteristics, such as sensitivity to small changes in initial conditions, long memory, non-stationarity, instability in catastrophic events, and the like. Complex signals exhibit complexity at different temporal and spatial scales, that is, they are multi-scaling. Earlier, using various

aspects of the manifestation of complexity, we also designed various measures of complexity that specifically respond to critical and crisis phenomena [1-4]. In this paper, we will examine how the Lyapunov stability of a complex system changes under conditions of financial crises [5]. As a tool, we choose the scale-dependent Lyapunov exponent (SDLE), the calculation features and the advantages of using which are described in [6, 7].

We briefly describe the idea and the formal foundations of the method, introduce new measures of complexity and illustrate their effectiveness with the example of the Dow Jones index. Let us have a single observation conducted at a discrete time interval Δt in the form of a time series $u_i(t)$ where $t = i \cdot \Delta t$. According to Taken's theorem, an equivalent phase trajectory that stores the structures of the original phase trajectory can be recovered from the time series by the time delay method: $\hat{x}(t) = (u_i, u_{i+\tau}, \dots, u_{i+(m-1)\tau})$, where m is the dimension of the attachment, τ is the time delay (the real time delay is defined as $\tau \cdot \Delta t$). After reconstructing the phase space, let us consider the ensemble of trajectories. Let us denote the initial distance between two close trajectories ε_0 , and their average distance at a time t and $t + \Delta t$ as ε_t and $\varepsilon_{t+\Delta t}$ respectively. Note that the classical algorithm for calculating the maximum Lyapunov exponent λ_1 is based on the assumption $\varepsilon_t \approx \varepsilon_0 e^{\lambda_1 t}$ and estimation λ_1 as $(\ln(\varepsilon_t - \varepsilon_0)) / t$. Depending on ε_0 , this property may not be true even for truly chaotic systems. To calculate the SDLE, we check whether the following inequality holds for a pair of vectors (V_i, V_j) :

$$\varepsilon_k \leq \|V_i - V_j\| \leq \varepsilon_k + \Delta \varepsilon_k, k = 1, 2, 3, \dots, \text{ where } \varepsilon_k \text{ and } \Delta \varepsilon_k \text{ there are arbitrarily chosen small values of distances, and}$$

$$\|V_i - V_j\| = \sqrt{\sum_{w=1}^m (x_{i+(w-1)L} - x_{j+(w-1)L})^2}. \quad \text{Geometrically, the last inequality defines a shell in high-dimensional space. Next, we}$$

investigate the dynamics of the same pairs of vectors (V_i, V_j) in the middle of the shell and perform averaging over the ensemble by indices i, j . Since the exponential or power functions are of the greatest interest, we assume that logging and averaging can be reversed. Finally, the following equation will look like:

$$\lambda(\varepsilon_t) = \frac{\left\langle \ln \|V_{i+t+\Delta t} - V_{j+t+\Delta t}\| - \ln \|V_{i+t} - V_{j+t}\| \right\rangle}{\Delta t}, \quad (1)$$

where t and Δt are the integer values of the sampling interval, the angle brackets correspond to the averaging over the indices i, j inside the shell, and

$$\varepsilon_t = \|V_{i+t+\Delta t} - V_{j+t+\Delta t}\| = \sqrt{\sum_{w=1}^m (x_{i+(w-1)L+t} - x_{j+(w-1)L+t})^2}. \quad (2)$$

Finally, note that

$$\Lambda(t) = \left\langle \ln \|V_{i+t} - V_{j+t}\| - \ln \|V_i - V_j\| \right\rangle \quad (3)$$

is called a time-dependent exponential curve. Because $\Lambda(t) = \ln \varepsilon_t - \ln \varepsilon_0$, we immediately see that SDLE corresponds to the local angle of the curve of the species: $\varepsilon_t = \varepsilon_0 \exp[\lambda(t)]$.

In the analysis of complex systems is also used an integral measure of complexity, which is calculated by the formula:

$$I = \ln \varepsilon_t = \ln \varepsilon_0 + \int_0^t \lambda(\varepsilon_t) dt. \quad (4)$$

Stock market indices, characterizing economic systems of varying degrees of complexity, differ in magnitudes $\Delta\lambda = \lambda_{\max} - \lambda_{\min}$, $\Delta\varepsilon = \varepsilon_{\max} - \varepsilon_{\min}$ and I . As an example, the integral measure I is calculated for a sliding window of 500 (top of the figure) and 1000 days for the daily values of the Dow Jones index (djia) for the period from January 1, 1983 to October 8, 2019 [8].

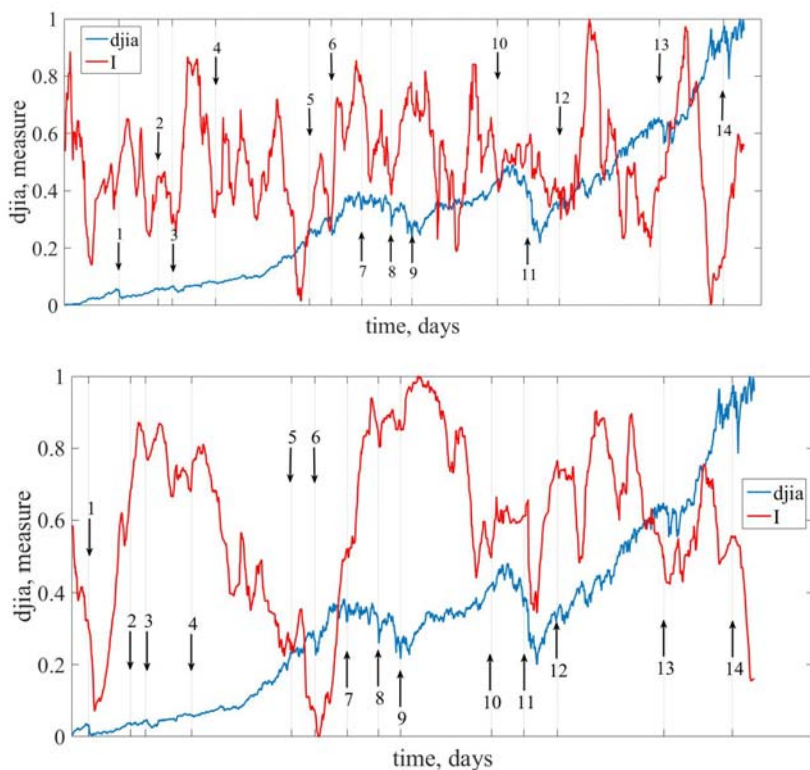


Fig. Comparative dynamics of the Dow Jones index and measures of integral complexity for windows of 500 and 1000 days. The arrows marked the crisis periods in accordance with the numbering in [3].

Obviously, this measure is a leading indicator of crisis phenomena. Resizing the sliding window allows you to separate the crises that are close in time.

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ANALYSIS AND MODELING OF TAX REVENUE DYNAMICS FOR UNITED TERRITORIAL COMMUNITIES

To date, the digitization of the economy has touched virtually every aspect of society life. This also applies to the activities of the united territorial communities (UTC). Due to the decentralization reform, since 2016, the annual increase of the revenues of the local budgets has been observed [1]. To the checking account of the territorial community, there are daily payments for the taxes and fees at the expense of which the UTC operates. Therefore, in order to plan their activities and ensure the functions of the UTC, it is an important question to study and analyze the dynamics of these revenues.

As a rule, each tax has different volume and a schedule of revenues (for example, the income tax is paid monthly, the land tax is paid once a year). Depending on the size and location of the community, the types of tax revenue may have several dozen items. Hence, modeling each individual tax is inappropriate, and the task of economists is to group taxes by the nature of their dynamics.