INVESTMENT STRATEGY MODELING ON INTERACTION OF SMALL ENTERPRISES

Economic-mathematical model of small enterprises functioning is considered in the paper. Dynamics of enterprises development providing investment mobilization is investigated.

Key words: economic-mathematical model, Laplace transformation, small enterprise

Introduction. Investment strategy modeling on interaction of small enterprises is one of actual problems of current economic relations (Samuelson, 1993), (Keynes, 1959).

First of all, production facilities as elements of economic system in dynamic models are considered (Bagrinovskiy, 1973). Such model gives the opportunity possibility to calculate the number of indicators (marginal utilities
of factors, elasticity of production by these factors etc.), that represent the
dynamic features of the system (Takayama, 1994). So, when considering the
dynamic models of economic systems with taking into account production
facilities there are situations with nonexistent leaps of such important economic
indicators as labor productivity and return on assets. The second feature of the
model is that the meaning of the indicator, which characterizes intensity of
production process (interval indicator), cannot be measured at any time, while
moment indicators-factors, which reflect the production condition, can be
measured at any time (Gruber, 1991). It follows, that it is necessary to consider
ways of production factors measuring concerning time interval for synthesis of
production facilities appropriate to specific object of production.

**Review and analysis of the previous research on the problem.** Over the
last few decades the main part of discussions concerning innovations has been
focused on studying the types of knowledge flows, that may help small
enterprises to use new technologies and possibilities of their transformation into
appropriate acquisition (Zahra, George, 2002), (Lazaric, Longhi, Thomas, 2008),
(Rammer, Czarnitzky, Spielkamp, 2009).

According to traditional theory of capital structure enterprises choose the
type of financing that minimize costs and maximize profits, connected with
different sources of external and internal capital (Titman, Wessels, 1988). Small
enterprises are informational, non-transparent and depended on investments.
Investors rely on soft information in lending of small business because the true
information of enterprise activity is restricted (Grunert, Norder, 2012).

**The unsettled part of the problem under study.** Despite the fact that a
great deal of studies have been devoted to interaction of two small enterprises
one of the current actual problems of Ukraine’s economy is investigating the
directions of investments use and investments features of small business. The
features influencing the dynamics of enterprise development in the whole as
well as the investment strategy have been analysed in the paper by applying economic-mathematical modeling of enterprise investment activity. For analytical solution it is necessary to show dynamic model of economic system as differential equation. For numerical describing of behavior we may use the form of production facilities with additional restriction, which provide the correct model behavior (Gruber, 1994), (Fuente, 2000), (Hands, 2004).

The purpose of the study is investigating small enterprises activity by means of economic-mathematical models with using operation calculus (Laplace transformation) depending on chosen investment strategies.

Main results of the study. Linear system as a complex of determined type links, which correspond to some economical processes, is considered.

Let us examine the mathematical model of economic process, beginning with the logical-structural chart (fig. 1).

![Logical-structural chart of strengthened link.](image)

Fig. 1 Logical-structural chart of strengthened link.

Source: designed by authors.

Let us denote centralized function of system as \( I(t) \) (for example, it is possible to consider investment function of enterprise), that becomes function of
production activity $X(t)$. After receiving revenue the enterprise deducts some part of revenue according actual standards that is invested in production with zero cycle of the beginning of production activity. It strengthens incoming investment process and influences outcoming function of production system $X_1(t)$.

Hence, this logical-structural chart may be represented in the form of single-factor dynamic model of production function (Fig. 2).

![Dynamic model of production function](image)

**Fig. 2 Dynamic model of production function.**

*Source: designed by authors.*

Let us consider the link in the production process, when capital investments are made only for put production facilities into operation.

If we examine the speed of production facilities function change or its intensity $f'(t)$ depending on capital investment intensity flow $i(t)$, operation lag $T$ (time period between incoming and outcoming processes, which have cause-consequence relation) and intensity of production facilities depreciation flow $a(t)$, we have:

$$f'(t) = i(t) - a(t)$$  \hspace{1cm} (1)
We know initial condition \( f(0) \) of production facilities status for \( t = 0 \). Relation of intensity function of depreciation flow and production facilities function will be:

\[
a(t) = \frac{1}{T} f(t).
\]

Let us denote \( \frac{1}{T} = n \) – depreciation rate, then we have

\[
a(t) = nf(t). \tag{2}
\]

Hence, from (2) and (1), we find:

\[
f''(t) = i(t) - nf(t). \tag{3}
\]

Let us solve differential equation of the first order with using the operation calculus. Due to these method linear differential equations with stable coefficients of desired function \( f(t) \) are implies to algebra equations concerning function \( F(p) \) (Martynenko, 1990).

We have operator equation:

\[
pF(p) - f(0) = I(p) - nF(p).
\]

Then we get:

\[
F(p) = \frac{I(p) + f(0)}{p + n} \tag{4}
\]

Equation (4) is named operator equation of production facilities intensity of capital investment intensity.

Let production function \( x(t) \) depend only on production facilities function \( f(t) \) (direct proportional dependence) and doesn’t depend on other factors, for
instance labor resources and their parameters, which have a little impact on final result. So, in the space of the original we get:

\[ x(t) = \mu f(t), \]

where \( \mu \) – capital productivity ratio.

Then appropriate operator equation will be:

\[ X(p) = \mu F(p). \]  \hspace{1cm} (5)

Substituting formula (4) in equation (5), we get:

\[ X(p) = \frac{\mu}{p+n} I(p) + \frac{\mu}{p+n} f(0). \]  \hspace{1cm} (6)

Then we can find the transfer function of production process link:

\[ W(p) = \frac{\mu}{p+n}. \]  \hspace{1cm} (7)

Hence,

\[ x(t) = W(p) * i(t). \]

Advisability of any activity is determined by its final result. Enterprise activity is evaluated by production efficiency, which in its turn depends on production efficiency, using and reproduction of fixed assets. They are the main “producers” and the result of this production. That’s why it is important to set quantitative dependence of production efficiency on using fixed assets efficiency (Forrester, 1978). Fixed assets are important fundamental indicator of economic potential of the state. That’s why an actual problem of existent enterprises is investigating fixed assets influence on production and the best conditions of its using. Production facilities become fixed assets, if they are used gradually, in parts, carrying its value for a long time, where this value is
accumulated. It is the main reason of including only those production facilities, which have long depreciation period (which are durable) into fixed assets.

The amount of production and sold products depend on the written-off value of fixed assets.

The amount of fixed assets is calculated in monetary terms or as value, which may be examined during the period \( t \in (0, T) \) (month, quarter, year). The written-off value of fixed productive assets is the value, which is calculated as difference between acquisition cost and depreciation value during the period of their operation. The written-off value increases, if capital investments grow, or decreases as a result of depreciation and production facilities retirement.

Capital investments consist of single-source facilities \( i(t) \) and withholding from revenue \( u(t) \). Revenue tax is imposed by the standard \( 0 < a < 1 \).

Then the functional structure of enterprise may be described as a model of two links: enterprise 1 (E1) and enterprise 2 (E2) in interaction.

Enterprise 2 denotes the process, where by investing in enterprise 1 the equity accumulation (excluding single-source capital investments) with standard withholding from the volume of sales \( a \) is taken place.

With the single-source capital investments equity impacts the production link 1, replacing the value of fixed productive assets and profit from sold products as the production function \( x(t) \).

Equation (1) considering equity \( aX(p) \) and equation (6) will be:

\[
X(p) = \frac{\mu}{p + n}(I(p) + aX(p)) + \frac{\mu}{p + n}f(0),
\]

or
\[ X(p) = \frac{\mu}{p - (a\mu - n)} I(p) + \frac{\mu}{p - (a\mu - n)} f(0) \].

Knowing that
\[ f(0) = \frac{x(0)}{\mu}, \]
we get
\[ X(p) = \frac{\mu}{p - (a\mu - n)} I(p) + \frac{1}{p - (a\mu - n)} x(0). \] (8)

Then
\[ x(t) = W(p) * i(t), \]
where
\[ W(p) = \frac{\mu}{p - (a\mu - n)}, \]
\[ w(t) = \mu e^{(a\mu-n)t}, \]
\[ x(t) = \int_0^t \mu e^{(a\mu-n)(t-\tau)} i(\tau) d\tau. \]

\( x(t) \) is production function depending only on production facilities.

Let us find the relationship between parameters of economic system of these enterprises for determining their transfer function as operator equation.

Production function \( X(p) \) is described by equation (8)
\[ X(p) = \frac{\mu}{p - (a\mu - n)} I(p) + \frac{x(0)}{\mu} \]
or, considering the action efficiency of E1, by equation (9)
\[ X(p) = \frac{\mu}{p - (a \mu - n)} (\eta X(p) + \frac{x(0)}{\mu}) \]  \quad (9)

Image \( I(p) \) of capital investment efficiency is the external investment. By assumption the external investment depends on operation efficiency of \( E_1 \), namely pro rata to production function:

\[ I(p) = \eta X(p), \quad \eta > 0. \]

The investment is sent to \( E_2 \) from coefficient \( \alpha \) or in other words \( \alpha I(p) \), and the remainder of investment is sent to \( E_1 \)

\[ (1 - \alpha) I(p) = \beta I(p), \quad \alpha + \beta = 1. \]

Then we have

\[ X_2(p) = \frac{\mu_2}{p - (a_2 \mu_2 - n_2)} \left( \alpha \eta X_2(p) + X_2^0 \right). \]

or

\[ X_2(p) = \frac{X_2^0}{p - (a_2 \mu_2 + \alpha \eta \mu_2 - n_2)}, \]

\[ X_2(p) \leftrightarrow x_2(t) = X_2^0 e^{(a_2 \mu_2 + \alpha \eta \mu_2 - n_2)t}, \quad X_2^0 = \text{const}. \]

Then – in \( E_1 \)

\[ X_1(p) = \frac{\mu_1}{p - (a_1 \mu_1 - n_1)} \left( \beta \eta X_1(p) + X_1^0 \right), \quad I = \eta X_2. \]

Substituting \( X_2(p) \) from equation (10) in equation (11), we get:

\[ X_1(p) = \frac{X_2^0}{p - (a_1 \mu_1 - n_1)} + \frac{\mu_1 \beta \eta X_2^0}{p - (a_1 \mu_1 - n_1)(p - (a_2 \mu_2 + \alpha \eta \mu_2 - n_2))}. \]
Using the method of undetermined coefficients, we get:

\[
X_1(p) = \frac{X_1^0}{p - (a_1\mu_1 - n_1)} + \\
\frac{1}{p - (a_1\mu_1 - n_1)} - \frac{1}{p - (a_2\mu_2 + \alpha\eta\mu_2 - n_2)} \frac{\mu_1\beta\eta X_2^0}{a_1\mu_1 - n_1 - a_2\mu_2 - \alpha\eta\mu_2 + n_2}.
\]

By the last equality we get the original

\[
x_1(t) = X_1^0 e^{b_1t} + \mu_1\beta\eta X_2^0 \left( \frac{1}{b_1 - b_2} (e^{b_2t} - e^{b_1t}) \right),
\]

where

\[
b_1 = a_1\mu_1 - n_1, \quad b_2 = a_2\mu_2 + \alpha\eta\mu_2 - n_2.
\]

If in equality (10)

\[
a_2\mu_2 + \alpha\eta\mu_2 - n_2 = 0,
\]

then

\[
a_2\mu_2 + \alpha\eta\mu_2 = n_2
\]

is the indifferent equilibrium condition, that is

\[
x_2(t) = x_2(0).
\]

E1 will be insolvent, if

\[
\alpha \geq \frac{n_2 - a_2\mu_2}{\eta\mu_2}.
\]

Condition

\[
\beta \leq 1 - \alpha \geq 1 - \frac{n_2 - a_2\mu_2}{\eta\mu_2}.
\]
is the condition of full bankruptcy.

By equation (10) we know, that transfer function will be

\[ W_2(p) = \frac{\mu_2}{p - (a_2\mu_2 + \alpha_2\mu_2 - n_2)}, \]

\[ X_2(p) = W_2(p) \frac{X_2(0)}{\mu_2}. \]

By equation (11) we know

\[ W_1(p) = \frac{\mu_1}{p - (a_1\mu_1 - n_1)}. \]

If there are no external investments, that is \( i(t) = 0 \), then equation (9) will be

\[ X(p) = \frac{\mu}{p - (a\mu - n)} \frac{X_0}{\mu}. \] (12)

The part of gross capital investments \( \alpha X(p) \) will be invested in production of \( E_2 \). If \( E_1 \) receives investments

\[ \nu a_2 x_2(t), \]

then \( E_2 \) receives

\[ (1 - \nu)a_2 x_2(t), \quad 0 < \nu < 1, \]

and the models of \( E_1 \) and \( E_2 \) will be (instead of \( a \) in equation (12) we use \( \nu a \)):

\[ X_2(p) = \frac{\mu_2}{p - (a_2\mu_2 + \alpha_2\mu_2 - n_2)} \frac{X_2^0}{\mu_2}. \] (13)

In formula (13) the transfer function will be
\[ W_2(p) = \frac{\mu_2}{p - (a_2 \mu_2 + \alpha \eta \mu_2 - n_2)}. \]

By equation (11) we get

\[ X_1(p) = \frac{\mu_i}{p - (a_i \mu_i - n_i)} \left( \frac{X_i^0}{\mu_i} - (1 - \nu) a_2 X_2(p) \right), \tag{14} \]

where transfer function is

\[ W_i(p) = \frac{\mu_i}{p - (a_i \mu_i - n_i)}. \]

The indifferent equilibrium condition of production in E1 fulfills providing

\[ \nu a_2 \mu_2 - n_2 = 0, \]

that is, if

\[ X(p) = X(0) = X_2^0 = \text{const}. \]

Hence, expanded reproduction in E1 will be, if

\[ \nu a_2 \mu_2 - n_2 > 0, \]

that is, the depreciation coefficient will be

\[ n_2 < \nu a_2 \mu_2. \tag{15} \]

From (13) i (14) we get, that production function equations of gross product for E1 and E2 are

\[ x_2(t) = X_2^0 e^{(a_2 \mu_2 + \alpha \eta \mu_2 - n_2) t}, \]

\[ x_1(t) = \frac{\mu_i (1 - \nu) a_2 x_2^0}{n_i - a_i \mu_i - n_2 + \nu a_2 \mu_2} \left( e^{(\nu_i (1 - \nu) a_2 - n_2) t} - e^{(\mu_i \eta_i - n_2) t} \right) + x_1^0 e^{(\mu_i \eta_i - n_2) t}. \]
As follows from (15), it is necessary to set the correlation of economic coefficients $\mu, \nu, a, n$, under which the production of $E_1$ doesn’t decrease \((n_2 = v a_2 \mu_2)\), but increases and expands the output and gross investments \((n_2 < v a_2 \mu_2)\).

**Conclusion.** Summing up the study we can make such conclusion:

1. It is shown that it is possible to analyze the model solutions in the dimension of frequency characteristics due to Laplace transformation. It enables us not to use inverse Laplace transformation and in its turn not to solve the most complicated problem of operation calculus – finding the original by the image.

2. It is derived that enterprise fixed assets dynamics is the linear combination of exponential functions. It depends on parameters, which characterize some enterprise possibilities, and demonstrates the correlation of economic coefficients, under which the output and gross investments increase.

**References.**


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